Multiple Cooper Minima in Quadrupole Photoionization

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Photoionization Parameters

- Photoionization Cross-section
- Angular Distribution Asymmetry Parameter
- Spin Polarization Parameters

\[ \propto \left| \langle \psi_f | \hat{T} | \psi_i \rangle \right|^2 \]
Differential cross-section for photoionization: \[
\frac{d\sigma}{d\Omega} \propto \left| \langle \psi_f | e^{ik\cdot \vec{r}} \hat{e} \cdot \vec{V} | \psi_i \rangle \right|^2
\]

Dipole Approx \( \lambda \gg r \): \( e^{ik\cdot \vec{r}} \approx 1 \) (\( h\nu \ll 5 \text{ keV} \))

Inclusion of Non-Dipole Terms: \( e^{ik\cdot \vec{r}} \approx 1 + ik\cdot \vec{r} \)

\[
\frac{d\sigma}{d\Omega} = \frac{\sigma_{nl}}{4\pi} \left[ 1 + \frac{\beta}{2} \left( 3\cos^2\theta - 1 \right) + (\gamma + \delta \cos^2\theta)\sin\theta\cos\phi \right]
\]

\( \beta \rightarrow \text{E1 Ang. dist. asym. param.} \)
(Interference of amplitudes in different E1 channels with each other)

\( \gamma, \delta \rightarrow \text{E2 Ang. dist. asym param.} \)
(Interference of E1 & E2 amplitudes)

\( \zeta = \gamma + 3\delta \)
Cooper Minimum (CM)

At CM: Transition matrix element ≈ 0.

\[
R_{l\pm 1} = \int_{0}^{\infty} P_{nl}(r) r P_{\epsilon,l\pm 1}(r) \, dr
\]

Matrix elements for p→d transitions in Ne, Ar and Kr

J. W. Cooper, Phys. Rev. 128, 2, 681 (1962)

At CM Photoionization parameters show certain features.
In general: Matrix element has a *single* Cooper minimum.

In Dipole Matrix elements Multiple Cooper minima occur:

- *In high Z atoms (Hg [1], Ra [2], Rn [3]) ; induced by interchannel coupling.*
- *In photoionization of excited state of atomic Cs 9d state [4].*

In the present study Multiple Cooper minima has been observed:

- *For Electric Quadrupole (E2) transitions.*
- *In ground state photoionization.*
- *Using Relativistic Random Phase Approximation (RRPA)[5], with minimum electron corrections included.*
Hg 5d E2: RRPA coupling channels only from 5d subshell (intrashell)

\[ \Delta l = 0, \pm 2 \]
\[ \Delta j = 0, \pm 1, \pm 2; \]
\[ j: 0 \leftrightarrow 0; j: \frac{1}{2} \leftrightarrow \frac{1}{2} \]

\[ \begin{align*}
5d_{3/2} & \rightarrow s \\
5d_{3/2} & \rightarrow d_{3/2} \\
5d_{3/2} & \rightarrow d_{5/2} \\
5d_{3/2} & \rightarrow g_{7/2} \\
5d_{5/2} & \rightarrow s \\
5d_{5/2} & \rightarrow d_{3/2} \\
5d_{5/2} & \rightarrow d_{5/2} \\
5d_{5/2} & \rightarrow g_{7/2} \\
5d_{5/2} & \rightarrow g_{9/2}
\end{align*} \]

2 Cooper minima at intrashell coupling of RRPA
Hg 5d E2: RRPA calculations at two levels of truncation

The position of cooper minima are found to be almost insensitive to interchannel coupling.
Kr 4p E2: RRPA coupling channels only from 4p subshell (Intrashell)

\[
\begin{align*}
4p_{1/2} & \rightarrow p_{3/2} \\
4p_{1/2} & \rightarrow f_{5/2} \\
4p_{3/2} & \rightarrow p_{1/2} \\
4p_{3/2} & \rightarrow p_{3/2} \\
4p_{3/2} & \rightarrow f_{5/2} \\
4p_{3/2} & \rightarrow f_{7/2}
\end{align*}
\]

2 Cooper minima at intrashell coupling of RRPA
Xe 5p E2: RRPA coupling channels only from 5p subshell (intrashell)

\[ 5p_{1/2} \rightarrow p_{3/2} \]
\[ 5p_{1/2} \rightarrow f_{5/2} \]

\[ 5p_{3/2} \rightarrow p_{1/2} \]
\[ 5p_{3/2} \rightarrow p_{3/2} \]
\[ 5p_{3/2} \rightarrow f_{5/2} \]
\[ 5p_{3/2} \rightarrow f_{7/2} \]

2 Cooper minima at Intrashell coupling of RRPA
Hg 5d E2: RRPA coupling channels only from 5d subshell (Intrashell)

1\textsuperscript{st} CM: \(\sim 250\) eV.
2\textsuperscript{nd} CM: \(\sim 1550\) eV.
**Kr 4p E2: RRPA coupling channels only from 4p subshell (Intrashell)**

1\textsuperscript{st} CM: \( \approx 95 \text{ eV} \).

2\textsuperscript{nd} CM: \( \approx 950 \text{ eV} \).
Xe 5p E2: RRPA coupling channels only from 5p subshell (Intrashell)

1\textsuperscript{st} CM: \(\sim 33\) eV.

2\textsuperscript{nd} CM: \(\sim 152\) eV.
Xe 5p E2 Xsc and Zeta: RRPA coupling channels only from 5p subshell

\[ \zeta_{5p} = \frac{\sigma_{5p_{1/2}}(\omega)\zeta_{5p_{1/2}}(\omega) + \sigma_{5p_{3/2}}(\omega)\zeta_{5p_{3/2}}(\omega)}{\sigma_{5p}}; \quad \sigma_{5p} = \sigma_{5p_{1/2}} + \sigma_{5p_{3/2}} \]

Total cross section for Xe 5p along with measurable quantity Zeta.
Conclusions:

- **Normally, only single cooper minimum is present in dipole cross sections.**

- **In high Z atoms, multiple cooper minima are induced by only interchannel coupling (intershell coupling).**

- **Cooper minima was found to occur in dipole photoionization of excited state of atomic Cs. (9d state)**

- **Multiple cooper minima are predicted in quadrupole cross sections of Hg 5d, Kr 4p and Xe 5p even with intrashell coupling.**

- **The position of cooper minima are found to be almost insensitive to intershell coupling.**

- **The closest minima are in the case of Xe 5p and are observed at around 33 eV and 152 eV.**

- **Splitting in cooper minima for spin-orbit split components is due to the fact that spin-orbit potential due to j=l+1/2 is repulsive and that due to j=l-1/2 is attractive.**
References:


