Scales of gravity, and tensor bounds on the hidden Universe

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Effective field theory and the scales of gravity

For any given momentum transfer, gravitational interactions have a strength set by a characteristic scale $M_*$...

... inferred from amplitudes calculated in an effective theory with strong coupling scale $M_{**}$. In pure gravity:

$$M_* = M_{**} = M_{pl} = 2.44 \times 10^{18} \text{ GeV}$$
Effective field theory and the scales of gravity

For any given momentum transfer, gravitational interactions have a strength set by a characteristic scale $M_*$

... inferred from amplitudes calculated in an effective theory with strong coupling scale $M_{**}$. In the presence of matter:

$$M_* \neq M_{**} \neq M_{pl}$$

Antoniadis, Patil '14, '15
Effective field theory and the scales of gravity

Consider some physical particle with mass $m$. Dvali, Redi ’07

Scatter a test particle off of some very heavy point mass.

When $\Delta x \sim \frac{\hbar}{mc}$, virtual pairs of these particles are created.

Positive/negative energy solutions attracted/repulsed from the source, effectively anti-screening it – gravity appears to have gotten stronger.
Effective field theory and the scales of gravity

- Consider some physical particle with mass $m$.
- Scatter a test particle off of some very heavy point mass.
- When $\Delta x \sim \frac{\hbar}{mc}$, virtual pairs of these particles are created.
- Positive/negative energy solutions attracted/repulsed from the source, effectively anti-screening it – gravity appears to have gotten stronger.
- What’s actually happening: each massive species contributes to lowering the scale where strong gravitational effects become important.
Effective field theory and the scales of gravity

- Consider the virtual effect of some massive particle $\phi$ with mass $m$.
- On a Minkowski background \cite{Dvali:2007hz}

\[
\sim \frac{1}{M_{\text{pl}}^4} \frac{1}{p^2} \langle T(-p)T(p) \rangle \frac{1}{p^2}
\]

- When $p^2 \gg m^2$, theory becomes conformal:

\[
\langle T(-p)T(p) \rangle \sim \frac{c}{16\pi^2} p^4 \log \frac{p^2}{\mu^2}
\]

- Central charge $c := N = \frac{4}{3} N_\phi + 8 N_\psi + 16 N_V$ \cite{Duff:1976pk}
Effective field theory and the scales of gravity

Consider the virtual effect of some massive particle $\varphi$ with mass $m$

On a Minkowski background \cite{Dvali, Redi '07}

$$\sim \frac{1}{M_{\text{pl}}^4} \frac{1}{p^2} \left\langle T(-p)T(p) \right\rangle \frac{1}{p^2}$$

When $p^2 \gg m^2$, theory becomes conformal:

$$\left\langle T(-p)T(p) \right\rangle \sim \frac{c}{16\pi^2} p^4 \log \frac{p^2}{\mu^2}$$

Free propagator $1/(p^2 M_{\text{pl}}^2)$; perturbative treatment fails at $p = M_{\text{pl}}/\sqrt{N} \equiv M_{**}$
Effective field theory and the scales of gravity

- Consider generalization to curved backgrounds:
  \[ S = \frac{M_{\text{pl}}^2}{2} \int d^4x \sqrt{-g} \, R + \int d^4x \sqrt{-g} \left[ c_1 R^2 + c_2 R^{\mu\nu} R_{\mu\nu} \right] + \ldots \]

- \( c_1, c_2 \) indices that count a spin weighted sum of the particle content \( \sim N \)

- Expansion breaks down when \( p^2 \sim M_{\text{pl}}^2/N \) or when \( R \sim M_{\text{pl}}^2/N \)

- e.g. during inflation, lets say we tried to calculate corrections to the graviton 2-pt function; \( h_{\mu\nu} = g_{\mu\nu} - g^{\mu\nu}_0 \)

- Leading term – \( S = \frac{M_{\text{pl}}^2}{8} \int d^4x \sqrt{-g^0} \left[ \dot{h}_{ij} \dot{h}_{ij} - \frac{1}{a^2} \partial_{k} h_{ij} \partial_{k} h_{ij} \right] \)

- Higher curvature contributions s.t. \( M_{\text{pl}}^2 \rightarrow M_{\text{pl}}^2 \left( 1 + c \frac{H^2}{M_{\text{pl}}^2} + \ldots \right) \)
Effective field theory and the scales of gravity

• Consider generalization to curved backgrounds:

\[ S = \frac{M_{\text{pl}}^2}{2} \int d^4x \sqrt{-g} \, R + \int d^4x \sqrt{-g} \left[ c_1 R^2 + c_2 R^{\mu \nu} R_{\mu \nu} \right] + \ldots \]

• Corollary – it is not possible to consistently infer a scale of inflation higher than

\[ H^2 \sim \frac{M_{\text{pl}}^2}{N} \]
Effective field theory and the scales of gravity

The strength of gravity $M_*$ (inferred e.g. from a Cavendish experiment) is an independent quantity.

(Can be $M_{\text{pl}}$ all the way up till $M_{**}$) Gasperini ’15

$N_*$ counts the number of contributing species with masses below the momentum transfer of the process in question.
Effective field theory and the scales of gravity

If species in question is a KK mode with mass $m_{KK}$, we have the additional tree-level exchange

$$\frac{1}{M_{pl}^2 p^2} \rightarrow \frac{1}{M_{pl}^2 p^2} + \frac{n}{M_{pl}^2 (p^2 + m_{KK}^2)}$$

In the regime $m_{KK}^2 \ll p^2 \ll M_{pl}^2/N$, strength of gravity is given by:

$$\frac{1}{M_{pl}^2 p^2} + \frac{n}{M_{pl}^2 p^2 \left(1 + m_{KK}^2 / p^2\right)} \rightarrow \frac{n+1}{M_{pl}^2 p^2}$$
Effective field theory and the scales of gravity

\[ \Delta \mathcal{L}_{\text{eff}} \sim \xi \phi^2 R \sim \xi \frac{\phi^2}{M_{\text{pl}}^2} T_{\mu}^{\mu} \]

- If species in question couples to the trace of the energy momentum tensor

- In the regime \( m^2_\phi \ll p^2 \ll M_{\text{pl}}^2/N \), expanding around \( \langle \phi \rangle = v \)

\[
\frac{1}{M_{\text{pl}}^2 p^2} \rightarrow \frac{1}{M_{\text{pl}}^2 p^2} + \frac{g^2}{M_{\text{pl}}^2 (p^2 + m^2_\phi)} \sim \frac{1+g^2}{M_{\text{pl}}^2 p^2}; \quad g^2 := \xi^2 v^2 / M_{\text{pl}}^2
\]

- \( M_\ast = M_{\text{pl}} / \sqrt{N_\ast} \); \( N_\ast \) a (process dependent) weighted index.
Hidden fields in the CMB, or nothing is still something

Del Rio, Durrer, Patil to appear

Can one convert the non-observation of spectral running in to constraints on hidden field content?

• Fields with masses less than H will be QM'ly excited.
• Even if they do not couple directly to the inflaton (i.e. only interaction is via gravity), they still have an effect on the interactions (after renormalizing background quantities).
• If there are a large number of them, could they overcome Planck and slow roll suppression of interactions, generate a non-trivial running?
Nothing is still something

\[ S = \frac{M_{\text{pl}}^2}{2} \int d^4x \sqrt{-g} R[g] - \frac{1}{2} \int d^4x \sqrt{-g} \left[ \partial_\mu \phi \partial^\mu \phi + 2V(\phi) \right] - \sum_{n=1}^{n_{\text{max}}} \frac{1}{2} \int d^4x \sqrt{-g} \left[ \partial_\mu \chi_n \partial^\mu \chi_n - m_n^2 \chi_n^2 \right] + \ldots \]

\[ ds^2 = -N^2 dt^2 + h_{ij}(dx^i + N^i dt)(dx^j + N^j dt) \]

\[ \phi(t, x) = \phi_0(t), \]

\[ h_{ij}(t, x) = a^2(t)e^{2\zeta(t,x)} \hat{h}_{ij}, \quad \hat{h}_{ij} = \exp[\gamma_{ij}] \]

\[ N = 1 + \alpha_1 \]

\[ N^i = \partial_i \theta + N^i_T, \quad w/ \partial_i N^i_T \equiv 0 \quad \alpha_1 = \frac{\dot{R}}{H} \quad \dot{\theta} = -\frac{\partial^2 R}{a^2 H} + c\dot{R} \]
Nothing is still something

\[ S_{2,\mathcal{R}} = M_{\text{pl}}^2 \int d^4 x \alpha^3 e \left[ \dot{\mathcal{R}}^2 - \frac{1}{\alpha^2} (\partial \mathcal{R})^2 \right] \]

\[ \epsilon := \frac{\ddot{\phi}_0}{2H^2 M_{\text{pl}}^2} \]

\[ S_{2,\chi} = \frac{1}{2} \int d^4 x \alpha^3 \left[ \dot{\chi}_n \dot{\chi}_n - \frac{1}{\alpha^2} \partial_i \chi_n \partial_i \chi_n - m_n^2 \chi_n^2 \right] \]

\[ S_{2,\gamma} = \frac{M_{\text{pl}}^2}{8} \int d^4 x \alpha^3 \left[ \dot{\gamma}_{ij} \dot{\gamma}_{ij} - \frac{1}{\alpha^2} \partial_k \gamma_{ij} \partial_k \gamma_{ij} \right] \]

\[ S_{3,\mathcal{R}X} = \frac{1}{2} \int d^4 x \left\{ \alpha^3 \dot{\chi}_n \dot{\chi}_n \left( 3\mathcal{R} - \frac{\dot{\mathcal{R}}}{H} \right) - 2\alpha^3 \dot{\chi}_n \dot{\partial_i \partial_i \chi}_n \right. \]

\[ \left. - \alpha^3 \left( \mathcal{R} + \frac{\dot{\mathcal{R}}}{H} \right) \frac{1}{\alpha^2} \partial_i \chi_n \partial_i \chi_n - \alpha^3 \left( 3\mathcal{R} + \frac{\dot{\mathcal{R}}}{H} \right) m_n^2 \chi_n^2 \right\} \]

\[ S_{3,\gamma\chi} = \frac{1}{2} \int d^4 x \alpha \left[ \gamma_{ij} \partial_i \chi_n \partial_j \chi_n \right] \]
Nothing is still something

\[ S_{3, \mathcal{R}_X} = \frac{1}{2} \int d^4x \left\{ a^3 \dot{\chi}_n \chi_n \left( 3 \mathcal{R} - \frac{\dot{\mathcal{R}}}{H} \right) - 2 a^3 \dot{\chi}_n \partial_i \dot{\chi}_n \partial_i \chi_n \right. \]
\[ - a^3 \left( \mathcal{R} + \frac{\dot{\mathcal{R}}}{H} \right) \frac{1}{a^2} \partial_i \chi_n \partial_i \chi_n - a^3 \left( 3 \mathcal{R} + \frac{\dot{\mathcal{R}}}{H} \right) m_n^2 \chi_n^2 \left\} \right. \]

\[ S_{3, \mathcal{R}_X} = \int d^4x a^3 \epsilon \left[ \frac{\mathcal{R}}{2} \left( \dot{\chi}_n \chi_n + \frac{1}{a^2} \partial_i \chi_n \partial_i \chi_n + m_n^2 \chi_n^2 \right) - \dot{\chi}_n \partial_i \chi_n \partial_i \partial^{-2} \mathcal{R} \right] \]

\( \epsilon \) is an order parameter – it book keeps the expansion
Nothing is still something

\[ S_{3,\mathcal{R}_\chi} = \int d^4 x \ a^3 \epsilon \left[ \frac{\mathcal{R}}{2} \left( \dot{\chi} \chi_n^\nu + \frac{1}{a^2} \partial_i \chi_n \partial_i \chi_n + m_n^2 \chi_n^2 \right) - \dot{\chi} \chi_n \partial_i \chi_n \partial_i \mathcal{R} \right] \]

\[ S_{3,\gamma\chi} = \frac{1}{2} \int d^4 x \ a \left[ \gamma_{ij} \partial_i \chi_n \partial_j \chi_n \right]. \]

\[ \langle \mathcal{R} \mathcal{R} \rangle \propto \frac{1}{\epsilon M_{\text{pl}}^2} \]

\[ \epsilon \]

\[ \epsilon^2 \]

\[ \sim \frac{N}{16\pi^2} \frac{H^2}{M_{\text{pl}}^4} \]
Nothing is still something

\[ S_{3,\mathcal{R}_X} = \int d^4 x \ a^3 \epsilon \left[ \frac{\mathcal{R}}{2} \left( \dot{\chi}_n \chi^n + \frac{1}{a^2} \partial_i \chi_n \partial_i \chi_n + m_n^2 \chi_n^2 \right) - \dot{\chi}_n \partial_i \chi_n \partial_i \partial^{-2} \mathcal{R} \right] \]

\[ S_{3,\gamma_X} = \frac{1}{2} \int d^4 x \ a \left[ \gamma_{ij} \partial_i \chi_n \partial_j \chi_n \right] . \]

\[ \sim \frac{\epsilon N^2}{(16 \pi^2)^2} \frac{H^4}{M_{p1}^6} \]

\[ \sim \frac{\epsilon N}{(16 \pi^2)^2} \frac{H^4}{M_{p1}^6} \]
Nothing is still something

\[ S_{3, \mathcal{R} \chi} = \int d^4 x \, a^3 \epsilon \left[ \frac{\mathcal{R}}{2} \left( \dot{\chi}_n \dot{\chi}^n + \frac{1}{a^2} \partial_i \chi_n \partial_i \chi_n + m_n^2 \chi_n^2 \right) - \dot{\chi}_n \partial_i \chi_n \partial_i \partial^{-2} \mathcal{R} \right] \]

\[ S_{3, \gamma \chi} = \frac{1}{2} \int d^4 x \, a \left[ \gamma_{ij} \partial_i \chi_n \partial_j \chi_n \right]. \]

FIG. 3: Two loop corrections to \( \langle \zeta \zeta \rangle \). Wavy lines denote the graviton propagator. The double sunset graphs dominate when \( N \gg 1/\epsilon \).

FIG. 4: Two loop corrections to \( \langle \gamma \gamma \rangle \), where here we only require \( N \gg 1 \) for the double sunset graphs to dominate.

... calculating the running of these quantities turns out to be rather non-trivial!
Nothing is still something

\[ S_{3, R_X} = \int d^4 x \, a^3 \epsilon \left[ \frac{\mathcal{R}}{2} \left( \dot{\chi}_n \dot{\gamma}^n + \frac{1}{a^2} \partial_i \chi_n \partial_i \chi_n + m^2 \chi_n^2 \right) - \dot{\chi}_n \partial_i \chi_n \partial_i \partial^{-2} \mathcal{R} \right] \]

\[ S_{3, \gamma_X} = \frac{1}{2} \int d^4 x \, a \left[ \gamma_{ij} \partial_i \chi_n \partial_j \chi_n \right] \]

\[ \langle \mathcal{O}(\tau) \rangle = \sum_{n=0}^{\infty} i^n \int_{\tau_0}^{\tau} d\tau_n \int_{\tau_0}^{\tau_{n-1}} d\tau_{n-1} \ldots \int_{\tau_0}^{\tau_2} d\tau_1 \langle [H_1(\tau_1), [H_1(\tau_2), \ldots [H_1(\tau_n), \mathcal{O}(\tau)] \ldots] \rangle \]

\[ \langle \mathcal{O}(\tau) \rangle = \langle 0_{in} | T_C \left[ \exp \left( -i \int H_1(\tau') d\tau' \right) \mathcal{O}(\tau) \right] | 0_{in} \rangle \]
(Interlude on loops in Inflation)

Weinberg in ‘05 calculated the one loop correction from a hidden field:

$$P_\zeta = \frac{H^2}{8\pi^2 M_{\text{pl}}^2 \epsilon} \left[ 1 - \epsilon \frac{4\pi}{15} \frac{H^2}{M_{\text{pl}}^2} \log \left( \frac{k}{\mu} \right) \right]$$

... which was subsequently verified by a host of authors.

Senatore and Zaldarriaga ‘09 – cannot be! Corrections must go like $\log \left( \frac{H}{\mu} \right)$ (seen from putting a hard cut-off in frequency).

Terms omitted in dimensional regularizing integrals (!)
Weinberg in ‘05 calculated the one loop correction from a hidden field:

\[ P_\zeta = \frac{H^2}{8\pi^2 M^2_{pl} \epsilon} \left[ 1 - \epsilon \frac{4\pi}{15} \frac{H^2}{M^2_{pl}} \log \left( \frac{k}{\mu} \right) \right] \]

Furthermore, Adshead, Easther and Lim pointed out vacuum selection prescription doesn’t always allow for the equivalence

\[ \langle \mathcal{O}(\tau) \rangle = \sum_{n=0}^{\infty} i^n \int_{\tau_0}^{\tau} d\tau_n \int_{\tau_0}^{\tau_{n-1}} d\tau_{n-1} \cdots \int_{\tau_0}^{\tau_2} d\tau_1 \langle [H_I(\tau_1), [H_I(\tau_2), \ldots [H_I(\tau_n), \mathcal{O}(\tau)] \ldots] \rangle \]

\[ \uparrow \]

\[ \langle \mathcal{O}(\tau) \rangle = \langle 0_{in} | T_C \left[ \exp \left( -i \int H_I(\tau')d\tau' \right) \mathcal{O}(\tau) \right] | 0_{in} \rangle \]
Therefore:

\[ P_\zeta = \frac{H^2}{8\pi^2M_{pl}^2\epsilon} \left[ 1 - \epsilon \frac{4\pi}{15} \frac{H^2}{M_{pl}^2} \log \left( \frac{H}{\mu} \right) \right] \]

SZ: correlation functions do not run as log k...

However \( H \equiv H_k \) – the Hubble rate when k’th mode `exits the horizon'.

Fixing the above at some pivot scale \( k_* \rightarrow \log (H_k/H_*) \)

\[ \log (H_k/H_*) \sim - \int_0^{N_k} \epsilon (N') dN'; \quad k = H_* e^{- \int_0^{N_k} (1+\epsilon)} \]

So that \( \log (H_k/H_*) = -\epsilon \log \left( \frac{k}{k_*} \right) \)

*otherwise no model of inflation would be eternal
Correlation functions do run, but much more weakly...

\[ P_\zeta = \frac{H^2}{8\pi^2 M_{\text{pl}}^2 \epsilon} \left[ 1 + N \epsilon^2 \frac{4\pi}{15} \frac{H^2}{M_{\text{pl}}^2} \log \left( \frac{k}{k_*} \right) \right] \left( \frac{k}{k_*} \right)^{n_s - 1 + \frac{1}{2} \frac{dn_s}{d \log k} \log \left( k/k_* \right)} \]

\[ P_\gamma = \frac{2H^2}{\pi^2 M_{\text{pl}}^2} \left[ 1 - \epsilon N \frac{3\pi}{10} \frac{H^2}{M_{\text{pl}}^2} \log \left( \frac{k}{k_*} \right) \right] \left( \frac{k}{k_*} \right)^{n_t + \frac{1}{2} \frac{dn_t}{d \log k} \log \left( k/k_* \right)} \]

Extra $\epsilon$ suppression, but *with opposite sign*.

* By criterion of CDNSZ, every model of inflation still eternal in spite of $\log k$ running...
Can in principle resum in the large $N$ limit...

\begin{align*}
P_\gamma & = \frac{\Delta \gamma \left( \frac{k}{k_\ast} \right) n_t(\epsilon_\ast, \dot{\epsilon}_\ast, \ldots)}{1 + \sim \bigcirc \sim} \\
P_\zeta & = \frac{\Delta \zeta \left( \frac{k}{k_\ast} \right)^{-1 + n_\sigma(\epsilon_\ast, \dot{\epsilon}_\ast, \ldots)}}{1 + \bigcirc \bigcirc} \\
\sim \bigcirc \sim & = \epsilon_\ast N \frac{3\pi H_*^2}{10 M_{pl}^2} \log \frac{k}{k_\ast} + \ldots \\
\bigcirc \bigcirc & = -c \epsilon_\ast^2 N \frac{3\pi H_*^2}{10 M_{pl}^2} \log \frac{k}{k_\ast} + \ldots
\end{align*}
Nothing is still something

\[ P_\gamma = \Delta_\gamma \left( \frac{k}{k_*} \right)^{-2\epsilon_* + \mathcal{O}(\epsilon^2)} \left[ 1 - \epsilon_* N \frac{3\pi H^2}{10 M_{pl}^2} \log \left( \frac{k}{k_*} \right) + \mathcal{O}(\epsilon^2) \right] \]

\[ n_t = -2\epsilon_1 - \epsilon_1 \lambda \quad n_t = -\frac{r_*}{8} \left( 1 + \frac{\lambda}{2} \right) \quad \lambda = \frac{12\pi}{5} \frac{N}{8} \frac{H^2}{M_{pl}^2} = \frac{3\pi^3}{20} N r_* \Delta \zeta \]

\[ \frac{1.41 \times 10^9}{r_*^2} \left( n_T + \frac{r_*}{8} \right) \approx N, \]

Therefore, if we can bound the quantity in the parenthesis from above to some significance by some amount \( \xi \) ... then

\[ N \lesssim \frac{1.44}{r_*^2} 10^9 \xi \]
Implications –

In the most optimistic case, if we detected \( r_\ast \sim 0.1 \) then if we could bound \( 10^{-4} \lesssim \xi \lesssim 10^{-2} \), then

\[
N \lesssim \xi \cdot 10^{11} \sim 10^7 - 10^9
\]

N.B. This is more competitive than the naïve strong coupling bound at \( r_\ast \sim 0.1 \) of \( N \leq 10^9 \)

SKA: nHz peak sensitivity – \( (k \sim 10^8 k_\ast \sim 10^5 \text{ Mpc}^{-1}) \)

If we detect tensors right at cosmic variance limit, then the bound \( > 10^{13} \ ...


Earlier solutions to the Hierarchy problem by invoking many copies of the Standard Model (up to \( N \sim 10^{32} \))