Notes on Ultrasonic Experiments

Material Properties and Sound Waves
Sound travels at different speeds in different materials. This is because the mass of the atomic particles and the force constants are different for different materials. The mass of the particles is related to the density of the material, and the interatomic force constant is related to the elastic constants of a material. The general relationship between the speed of sound in a solid and its density and elastic constants is given by the following equation:

\[ V = \sqrt{\frac{C \mu}{\rho}} \]

Where \( V \) is the speed of sound, \( C \) is the elastic constant, and \( \rho \) is the material density. This equation may take a number of different forms depending on the type of wave (longitudinal or shear) and which of the elastic constants that are used. The typical elastic constants of a materials include:

- Young's Modulus, \( E \): a proportionality constant between uniaxial stress and strain.
- Poisson's Ratio, \( \nu \): the ratio of radial strain to axial strain
- Bulk modulus, \( K \): a measure of the incompressibility of a body subjected to hydrostatic pressure.
- Shear Modulus, \( G \): also called rigidity, a measure of a substance's resistance to shear.
- Lame's Constants, \( \lambda \) and \( \mu \): material constants that are derived from Young's Modulus and Poisson's Ratio.

When calculating the velocity of a longitudinal wave, Young's Modulus and Poisson's Ratio are commonly used. When calculating the velocity of a shear wave, the shear modulus is used. It is often most convenient to make the calculations using Lame's Constants, which are derived from Young's Modulus and Poisson's Ratio.

It must also be mentioned that the subscript \( ij \) attached to \( C \) in the above equation is used to indicate the directionality of the elastic constants with respect to the wave type and direction of wave travel. In isotropic materials, the elastic constants are the same for all directions within the material. However, most materials are anisotropic and the elastic constants differ with each direction. For example, in a piece of rolled aluminum plate, the grains are elongated in one direction and compressed in the others and the elastic constants for the longitudinal direction are different than those for the transverse or short transverse directions.
Examples of approximate compressional sound velocities in materials are:

- 1020 steel - 0.589 cm/microsecond
- Cast iron - 0.480 cm/microsecond.

Examples of approximate shear sound velocities in materials are:

- 1020 steel - 0.324 cm/microsecond
- Cast iron - 0.240 cm/microsecond.

### Comparison of Different Atomic Bonds

<table>
<thead>
<tr>
<th>Bond Type</th>
<th>Typical Solids</th>
<th>Bond Energy (eV/atom)</th>
<th>Melt. Temp. (°C)</th>
<th>Elastic Modulus (GPa)</th>
<th>Density (g cm⁻³)</th>
<th>Typical Properties</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ionic</td>
<td>NaCl, (rock salt)</td>
<td>3.2</td>
<td>801</td>
<td>40</td>
<td>2.17</td>
<td>Generally electrical insulators. May become conductive at high temperatures. High elastic modulus. Hard and brittle but cleavable.</td>
</tr>
<tr>
<td></td>
<td>MgO, (magnesia)</td>
<td>1.0</td>
<td>2852</td>
<td>250</td>
<td>3.58</td>
<td>Thermal conductivity less than metals. Electrical conductor. High elastic modulus.</td>
</tr>
<tr>
<td></td>
<td>Mg</td>
<td>1.1</td>
<td>650</td>
<td>44</td>
<td>1.74</td>
<td>Good electrical insulator. Moderate thermal conductivity, though diamond has exceptionally high thermal conductivity.</td>
</tr>
<tr>
<td>Covalent</td>
<td>Si</td>
<td>4.0</td>
<td>1410</td>
<td>190</td>
<td>2.33</td>
<td>Large elastic modulus. Hard and brittle. Diamond is the hardest material.</td>
</tr>
<tr>
<td></td>
<td>C</td>
<td>7.4</td>
<td>3550</td>
<td>827</td>
<td>3.52</td>
<td>Good electrical insulator. Low elastic modulus. Some ductility.</td>
</tr>
<tr>
<td>van der Waals:</td>
<td>PVC, (polymer)</td>
<td>0.2</td>
<td>212</td>
<td>4</td>
<td>1.3</td>
<td>Low elastic modulus. Electrical insulator. Poor thermal conductivity.</td>
</tr>
<tr>
<td>Hydrogen bonding</td>
<td>H₂O, (ice)</td>
<td>0.9</td>
<td>0</td>
<td>9.1</td>
<td>0.917</td>
<td>Poor thermal conductivity. Large thermal expansion coefficient.</td>
</tr>
<tr>
<td>van der Waals:</td>
<td>Crystalline</td>
<td>0.09</td>
<td>-189</td>
<td>8</td>
<td>1.8</td>
<td>Low elastic modulus. Electrical insulator. Poor thermal conductivity.</td>
</tr>
<tr>
<td>Induced dipole</td>
<td>Argon</td>
<td>0.09</td>
<td>-189</td>
<td>8</td>
<td>1.8</td>
<td>Low elastic modulus. Electrical insulator. Poor thermal conductivity.</td>
</tr>
</tbody>
</table>

### Acoustic Impedance

Sound travels through materials under the influence of sound pressure. Because molecules or atoms of a solid are bound elastically to one another, the excess pressure results in a wave propagating through the solid. The **acoustic impedance** \( Z \) of a material is defined as the product of its density \( \rho \) and acoustic velocity \( V \).

\[
Z = \rho V
\]

Acoustic impedance is important in

1. the determination of acoustic transmission and reflection at the boundary of two materials having different acoustic impedances.
2. the design of ultrasonic transducers.
3. assessing absorption of sound in a medium.
Ultrasonic waves are reflected at boundaries where there is a difference in acoustic impedances \((Z)\) of the materials on each side of the boundary. (See preceding page for more information on acoustic impedance.) This difference in \(Z\) is commonly referred to as the impedance mismatch. The greater the impedance mismatch, the greater the percentage of energy that will be reflected at the interface or boundary between one medium and another.

The fraction of the incident wave intensity that is reflected can be derived because particle velocity and local particle pressures must be continuous across the boundary. When the acoustic impedances of the materials on both sides of the boundary are known, the fraction of the incident wave intensity that is reflected can be calculated with the equation below. The value produced is known as the reflection coefficient. Multiplying the reflection coefficient by 100 yields the amount of energy reflected as a percentage of the original energy.

\[
R = \left(\frac{Z_2 - Z_1}{Z_2 + Z_1}\right)^2
\]

Since the amount of reflected energy plus the transmitted energy must equal the total amount of incident energy, the transmission coefficient is calculated by simply subtracting the reflection coefficient from one.

**Refraction and Snell's Law**

Refraction takes place at an interface due to the different velocities of the acoustic waves within the two materials. The velocity of sound in each material is determined by the material properties (elastic modulus and density) for that material.

Snell's Law describes the relationship between the angles and the velocities of the waves. Snell's law equates the ratio of material velocities \(V_1\) and \(V_2\) to the ratio of the sine's of incident \((\theta_1)\) and refracted \((\theta_2)\) angles, as shown in the following equation.

\[
\frac{\sin \theta_1}{V_{L1}} = \frac{\sin \theta_2}{V_{L2}}
\]

Where:
- \(V_{L1}\) is the longitudinal wave velocity in material 1.
- \(V_{L2}\) is the longitudinal wave velocity in material 2.
In the diagram, the incident longitudinal wave ($V_{L1}$), a reflected longitudinal wave ($V_{L1'}$) and refracted longitudinal waves ($V_{L2}$) are shown. The reflected wave is reflected at the same angle as the incident wave because the two waves are traveling in the same material, and hence have the same velocities.

**Mode Conversion**
When sound travels in a solid material, one form of wave energy can be transformed into another form. For example, when a longitudinal wave hits an interface at an angle, some of the energy can cause particle movement in the transverse direction to start a shear (transverse) wave. Mode conversion occurs when a wave encounters an interface between materials of different acoustic impedances and the incident angle is not normal to the interface.

Snell's Law holds true for shear waves as well as longitudinal waves and can be written as follows.

\[
\frac{\sin \theta_1}{V_{L1}} = \frac{\sin \theta_2}{V_{L2}} = \frac{\sin \theta_3}{V_{S1}} = \frac{\sin \theta_4}{V_{S2}}
\]

Where:
- $V_{L1}$ is the longitudinal wave velocity in material 1.
- $V_{L2}$ is the longitudinal wave velocity in material 2.
- $V_{S1}$ is the shear wave velocity in material 1.
- $V_{S2}$ is the shear wave velocity in material 2.

It can be seen that when a wave moves from a slower to a faster material, there is an incident angle which makes the angle of refraction for the longitudinal wave 90 degrees. This is known as the first critical angle and all of the energy from the refracted longitudinal wave is now converted to a surface following longitudinal wave. This surface following wave is sometime referred to as a creep wave and it dampens out very rapidly with increasing distance.

Beyond the first critical angle, only the shear wave propagates into the material. For this reason, most angle beam transducers use a shear wave so that the signal is not complicated by having two waves present. In many cases there is also an incident angle that makes the angle of refraction for the shear wave 90 degrees. This is known as the second critical angle and at this point, all of the wave energy is reflected or refracted into a surface following shear wave or shear creep wave. Slightly beyond the second critical angle, surface waves will be generated.
Normal Beams
When a transducer (usually a piezoelectric sensor) is placed in contact with a material, longitudinal waves are transmitted into the material. If the same transducer is used to receive the reflected/scattered waves, the configuration is termed “pulse-echo” method. If a separate transducer is used to receive the reflected/scattered waves, the configuration is termed “pitch-catch” method.

Angle Beams
Wedges are typically used to introduce a refracted shear wave into the test material. The angle of refraction is only accurate for a particular material, which is usually steel.

Signal Characteristics
Normally, when a signal is measured with an oscilloscope, it is viewed in the time domain (vertical axis is amplitude or voltage and the horizontal axis is time). For many signals, this is the most logical and intuitive way to view them.
The frequency domain display shows how much of the signal's energy is present as a function of frequency. For a simple signal such as a sine wave, the frequency domain representation does not usually show us much additional information. However, with more complex signals, such as the response of a broad bandwidth transducer, the frequency domain gives a more useful view of the signal.

Fourier theory says that any complex periodic waveform can be decomposed into a set of sinusoids with different amplitudes, frequencies and phases. The process of doing this is called Fourier Analysis, and the result is a set of amplitudes, phases, and frequencies for each of the sinusoids that makes up the complex waveform. Adding these sinusoids together again will reproduce exactly the original waveform. A plot of the frequency or phase of a sinusoid against amplitude is called a spectrum.

**Lab activity:**
You are provided solid media of several types (metal, granite, wood, glass etc). You are to determine the longitudinal and shear wave speeds in these materials and estimate the various elastic moduli, Poisson ratio of these materials. The longitudinal wave speed is to be determined using the pulse-echo mode of operation. The shear wave speed is to be determined using the “through transmission” mode of operation.

1. With normal beam transduction in the pulse-echo method, estimation of longitudinal wave speed from distance and time of flight data.
2. With two angle beam transducers in the “through-transmission” mode, estimation of shear wave speed from distance and time of flight data.
3. With angle beam transducers in the pitch-catch mode, checking for mode conversion.
4. Estimation of elastic moduli and Poisson ratio with longitudinal and shear wave speeds (given density)

You are to familiarize yourself with Snell’s laws associated with plane waves, critical angle calculations, acoustic impedances, and impedance mismatch.

**Useful information:**
- Longitudinal wave speed in wedge material: 2700 m/s
- Longitudinal wave speed in steel: 5890 m/s
- Shear wave speed in steel: 3240 m/s
- Angle of refraction produced by the wedge in steel: 45°
Generation of Ultrasound

Remember the tuning fork? It was used to produce sound waves at a specific frequency. The tuning fork was made to “resonate” at a specific frequency. You also recall that the sound was heard long after the tuning fork was struck. In other words, the tuning fork was lightly damped.

To generate ultrasound, typically a piezo-electric crystal (such as quartz that you may have found in quartz clocks) is used. The dimensions of the crystal are chosen to let it resonate at a specific frequency.

You will see in the lab transducers that have these resonating crystals and something more.

![Diagram of transducer components](image)

**Figure 1**

On the left of Figure 1, we see a typical transducer with mechanisms for damping which will result in a short duration sound pulse to be generated. On the right of Figure 1, a schematic shows the sound generation (damped quartz crystal on the left), sound propagation in a medium (water in this case) and sound reception (another damped quartz crystal). The sequence of what happens when a short voltage pulse is applied to a damped crystal is indicated alongside (top trace). The sound pressure pulse that propagates in water (the medium in this case) is shown as the middle trace. What we see on the oscilloscope is what is depicted as the received voltage (bottom trace).