MAGNETIC SUSCEPTIBILITY USING A GOUY BALANCE

AIM:

To determine the magnetic susceptibility of a paramagnetic sample by measuring the force exerted on the sample by a magnetic field gradient

Introduction:

The electron has an intrinsic angular momentum characterized by a quantum number $\frac{1}{2}$. The quantized angular momentum of a free electron is $S = \hbar \sqrt{\frac{1}{2} (\frac{1}{2} + 1)}$. The intrinsic angular momentum can be crudely visualized as an intrinsic current loop which produced a magnetic moment. Thus each electron in the universe is a tiny magnet. (You will learn more about this in your quantum mechanics and atomic physics courses. Do not visualize the electron like a spinning top. Spin angular momentum is a truly intrinsic fundamental property of the electron).

You maybe familiar with the filling of atomic shells for a many-electron atom (Hunds rule, Aufbau principle etc). Configurations in which the shell is fully filled results in zero net spin quantum number and net orbital angular momentum quantum number. Such atoms, eg. Argon, Neon etc., do not have a net magnetic moment and referred to as diagmagents. Atoms which do not have fully filled outer shell possess a net magnetic moment (eg. Fe, Ni etc.). A collection of such atoms which forms a gas, liquid of solid is magnetic since in the presence of an applied field the tiny moments can swing in the direction of the field. This behavior is affected by the temperature of the sample (more on this in your Statistical physics course). If the tiny moments do not ‘interact’ with each other the materials is referred to as a paramagnet. Interaction among moments results in ferromagnets or antiferromagnets (You will read about the origins of ferromagnetism in your statistical physics, condensed matter and atomic physics courses.)

Consider a paramagnet at room temperature subject to a magnetic field $H$. An obvious quantity of interest is the magnetization, $M$ (magnetic moment ($m_{\mu}$) per unit volume). The magnetic susceptibility ($\chi$) is defined as ratio of the magnetization to the

---

1 Just to get you interested we mention as astonishing fact: ferromagnetism arises due to a combination of the Coulomb repulsion between electrons and how it is influenced by Pauli’s exclusion principle. Ferromagnetism cannot be modeled by considering dipole-dipole interactions!
applied magnetic field. The magnetization of a magnetic sample (paramagnet or ferromagnet) can be measured by a variety of methods a few of which you will be exposed to in the lab courses.

In this experiment we focus on the measurement of the force exerted on the sample by magnetic field gradient. The magnetic moment can also be measured in terms of an induced voltage in an electrical circuit (How this can be achieved?).

Consider a solid in which each electron has an orbital angular momentum characterized by the quantum number, \( L \), in addition to the spin angular momentum. Assuming spin-orbit coupling the total angular momentum quantum number is characterized by \( J \). The total magnetic moment of the atom is given by \( m_\mu = g \mu_B J \), where \( g \) is the Landé \( g \) factor of the atom and \( \mu_B \) is the Bohr magneton (\( \mu_B = eh/2m \)).

The difference in magnetic potential energy per unit volume between a substance of permeability \( \mu \) and the displaced medium, usually air of permittivity \( \mu_o \), is\(^2\)

\[
U = \left( \frac{H \cdot B}{2} \right)_{\text{air}} - \left( \frac{H \cdot B}{2 \mu_o} \right)_{\text{sample}} = \frac{\mu_o H^2}{2} - \frac{\mu_B (1 + \chi_m) H^2}{2} = -\mu_o \frac{H^2}{2} \chi_m
\]  

(1)

Here \( \chi_m \) is the magnetic susceptibility. Which **for small magnetic fields**\(^3\) is defined as

\[
\chi_m = \frac{M}{H}, \text{ where } M \text{ is the magnetization.}
\]

When a magnetic field, \( B \), is applied the energy changes by an amount

\[
E = -m_\mu \cdot B = -VM \cdot B
\]  

(2)

where \( V \) is the volume of the sample. Connect equations (1) and (2).

If there is a gradient in the magnetic field along the \( z \) direction, the sample experiences a force per unit volume given by (assuming \( \chi_m \) is uniform throughout the sample)

\[
f = -\frac{dU}{dz} = \frac{\mu_o \chi_m}{2} \frac{d}{dz} \left( \frac{H^2}{2} \right)
\]

(3)

\(^2\) Consult a book on electromagnetic theory to understand the origin of this expression.

\(^3\) You will learn the more general expression in the statistical physics course.
Thus the force is produced by the non-uniform field. A simple way to produce a field gradient is to use a specimen in the form of a long rod or tube filled with power or liquid placed between the pole pieces of an electromagnet which produced a uniform magnetic field as shown in the figure.

Since the length over which the uniform magnetic field is produced is much smaller than the sample length, the sample experiences a field gradient. In this case the total force is given by

\[
F = \int_{l_1}^{l_2} fAdz = A \frac{\mu_0 \chi_m}{2} (H_1^2 - H_2^2) \approx A \frac{\mu_0 \chi_m}{2} H_1^2
\]

where \(l_1, l_2\) is the length of the sample tube and \(A\) its area of cross-section and \(H_1\) and \(H_2\) are the magnetic field strengths along the \(z\) axis as indicated in the above figure. Now think of a physical balance in which the sample tube is hung from one side and is subject to a magnetic field. The other side has the standard weight pan as shown in figure 1. When the magnetic field is zero the weight of the sample is determined by the physical balance and is entirely due to gravity. When the field is switched on the magnetic force manifests as an apparent weight change of the sample (will the weight increase or decrease? How is this related to magnetic nature of the sample?). The force can easily be measured in terms of a weight by determining the new weight of the sample. This is known as a Guoy balance after the French physicist Louis Georges Gouy. **A modern version of the Guoy balance available in the laboratory uses a digital balance instead of a physical balance.**

Are you justified in neglecting \(H_2\)? If you keep decreasing the amount of power you take at what height does the method fail? Verify this.

**APPARATUS:**
The Guoy balance, the powder specimen (FeCl₂ or Fe₂SO₄) in a glass tube, dc power supply for the magnet.

**PROCEDURE:**
The electromagnet is energized by a DC power supply. The variable magnetic field is provided by the wedge-shaped pole-pieces. The entire electromagnet is housed inside a wooden casing. The distance between the pole-pieces can be varied by means of a handle on top of the wooden casing. A digital balance is placed which carries a hook at the bottom for suspending the glass tube containing the material (FeCl$_2$, or Fe$_2$SO$_4$). The magnetic field between the pole pieces can be varied by changing the current through the coils using a DC power supply. The magnetic field corresponding to the current through the coils can be determined using a Gaussmeter (How does this work?).

1. Zero-adjust the digital balance.
2. Determine the area of cross-section of the tube. Suspend the empty glass tube as shown in Fig.1 and find its weight in zero magnetic field.
3. Using the D.C. power supply, vary the current from 0 to 3.5 A in steps of 0.2 A and in each case find the weight of the empty glass tube (Why do this?)
4. Fill the tube with the given sample (say FeCl$_2$) to about 3/4ths of the tube. Find the weight of the filled glass tube to an accuracy of 10 mg., in zero magnetic field.
5. As before, find the weight of the filled glass tube in different applied magnetic fields (both for the increasing and decreasing fields). (Why do this? When can you expect a difference in readings taken for increasing and decreasing fields)
6. Repeat the experiment with one or two more substances.

When the magnetic force is measured in terms of weight equation (3) becomes

\[ mg = A \frac{H_0 \chi_m}{2} \left( H_1^2 - H_2^2 \right) \approx A \frac{H_0 \chi_m}{2} H_1^2 \quad (?) \text{ can you make this approximation} \]  \hspace{1cm} (5)

Plot a graph between \( m \) and \( H^2 \) to determine the susceptibility. This gives the susceptibility of a given volume. Compute the molar susceptibility of the sample. What is smallest susceptibility change that can be measured in the instrument? Is this sufficient to detect diamagnetism? Can you use this method for ferromagnets?

Are there gradients in the other two perpendicular directions? When can we neglect their effect?
IMPORTANT INSTRUCTIONS:

1. Reduce the current through the coils to zero slowly and then switch off the power supply.
2. DO NOT change the distance between the pole-pieces.
3. Switch off the digital balance. The glass tube is taken out of the balance and kept on the table. The power supply to the electro magnet is also turned off.

Tables
Table I

<table>
<thead>
<tr>
<th>S.No.</th>
<th>Wt. of the empty glass tube (gm)</th>
<th>Current through the coils (A)</th>
<th>Magnetic field (Gauss)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table II

<table>
<thead>
<tr>
<th>S.No.</th>
<th>Wt. of the substance (gm)</th>
<th>Current through the coils (A)</th>
<th>Magnetic field (Gauss)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Fig. 1. The conventional Guoy balance. NS is an electromagnet with power supply and AB is the experimental glass tube. **In your experiment the physical balance will be replaced by an accurate digital balance.**

FURTHER READING:

1. Think of a way in which the susceptibility could be measured by holding the sample fixed and working with moving magnets (Known as Evan’s design).
2. Think of other areas when magnetic forces play a role.