PHOTOREFRACTIVE PHASE REVERSAL SPECKLE PHOTOGRAPHY FOR EVALUATION OF IN-PLANE DISPLACEMENT

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ABSTRACT

A real-time photorefractive phase shifting read-out system is developed for quantitative evaluation of fringe patterns obtained in speckle photography. The optical system consists of a phase reversal speckle photography (PRSP) arrangement with a narrow laser beam as an illuminating source and the photorefractive crystal, BaTiO$_3$ as a recording medium. The phase (contrast) reversal is accomplished by varying the pressure within an air-filled quartz cell inserted in the pump beam (reference beam) of a two-beam coupling configuration. The pump beam interferes with the imaged speckle pattern (object beam), and creates a dynamic grating inside the BaTiO$_3$ crystal. In this paper, it is shown that phase reversal is achieved when a $\pi$- phase shifted speckle pattern overlaps on a unshifted speckle pattern at the observation plane. A detailed speckle pattern analysis is carried out by employing the PRSP technique and the experimental results on a diffuse surface subjected to rotation in its own plane using four-frame phase shifting technique are presented. With the slow response time of the BaTiO$_3$ crystal, and by skillful management of the exposure times, the proposed technique is a simple, attractive and alternative method for fringe analysis.

Keywords: Speckle Metrology, Photo-refractive materials, Two-beam coupling, Optical techniques, Phase shifting,
1. INTRODUCTION

Speckle correlation technique, also known as *speckle photography* is extensively used for non-contact determination of in-plane components of deformation of a diffusely reflecting object [1]. The speckle photography makes use of the random collection of bright and dark speckles formed in space, when a diffuse surface is illuminated by a coherent light. It is essentially consists of recording and analyzing the positional shift of the speckles generated before and after the deformation of the object. The resulted speckle correlation fringe patterns at the filtering plane of the recorded speckle pattern (*specklegram*), reveal the information pertaining to the nature and magnitude of deformation undergone by the object. As speckle photography is based on the positional shift of the speckles, it offers reduced sensitivity making suitable for measuring relatively large displacement measurements and hence found wide applications in solid mechanics, fluid mechanics and velocimetry [1].

Traditionally, photographic plates are employed as a recording medium, and point-wise or Fourier filtering methods are adopted to map the desired information on displacement. The emergence of photorefractive materials with their unique properties of higher resolution, low intensity operation and the capability of real-time response has been shown to hold promise in several photonic application such as optical data storage, dynamic holography and speckle metrology [2]. Speckle photography first demonstrated for real-time displacement, tilt and vibration analysis with a BSO crystal by Tiziani et.al.[3]. An Argon Ion laser was used to create the dynamic grating of the speckle pattern inside the crystal and a low power He-Ne laser reads the grating to obtain the fringe pattern. Since then various optical arrangements have been emerged in literature for real-time observation of speckle correlation fringes [4-12]. Liu et al. [6] proposed a novelty filtering technique in a slow response crystal such as BaTiO$_3$. In this filtering technique, a shifted speckle pattern is compared with the un-shifted original speckle pattern and the difference between these two is shown as a fringe pattern. Both amplification and de-amplification in a two-beam coupling arrangement was studied by changing the orientation of the c-axis crystal [7,8]. That is, if the phase shift between the index grating and the interference pattern is $+\pi/2$ gives a bright speckle background due to the amplification, whereas $-\pi/2$, yields a dark speckle back in the de-amplification mode of operation. A two-beam coupling configuration in BaTiO$_3$ crystal was recently exploited for the measurement of crystal parameters such as diffraction efficiency, two-beam coupling growth (grating formation) and eraser rates [7,8,11]. It is also emphasized that the slow response time of the BaTiO$_3$ crystal can be used to perform dynamic...
holography by skillful management of the exposure times. The slow response takes a finite time to reach its steady state, and the dynamic grating (corresponding to the initial state of the object) created inside the crystal takes fairly long time to decay. A method to record the various loading states of the object or the phase shifted interferograms for measurements within the decay time is also demonstrated [12]. A similar procedure is also adopted in the present analysis.

A real-time phase reversal speckle photography (PRSP) using a conventional two-beam coupling arrangement in BaTiO$_3$ crystal [10] is illustrated in this paper. The phase (contrast) reversal is accomplished by varying the pressure within an air-filled quartz cell inserted in the pump beam. It is shown that a phase reversal is achieved when a $\pi$-shifted speckle pattern overlaps on an un-shifted speckle pattern at the observation plane. This phenomena is exploited for phase shift calibration and quantitative evaluation of the in-plane displacement. The experimental results using a four-frame phase shifting technique on a diffuse surface subject to in-plane rotation are presented.

2. OPTICAL ARRANGEMENT

The schematic of the optical arrangement of PRSP is shown in Fig.1. An unexpanded laser beam from a 10 mW He-Ne laser is divided into two beams by a variable beam splitter (VBS). A thin diffuse object (DG) that is situated at a distance $l_1$ from the crystal is illuminated with one of these beams. The scattered speckle pattern generated from the diffuse surface, which is in the transmitting geometry (object beam) is imaged onto a BaTiO$_3$ crystal (5x5x5 mm$^3$) by lens $L_1$. The magnification factor, $M$ at the crystal plane is $l_2/l_1$. An unexpanded plane wave (pump beam) making an angle $2\theta$ with respect to the optical axis of the object beam is added at the crystal. To accomplish the phase reversal in the interferometer, an air-filled quartz cell is inserted in the pump beam. The pressure inside the cell is varied with the help of a pressure pump and it is monitored using a precision pressure gauge. An additional lens $L_2$ is placed at a distance, $f$, focal length of the lens $L_2$; and the speckle patterns are observed and recorded by using a ground glass screen (GG).
3. SPECKLE PATTERN ANALYSIS AND PHASE SHIFT CALIBRATION

3.1 Theory

In this two-beam coupling configuration, a weak scattered imaged object beam of intensity $I_o$ at the crystal plane interferes with a strong pump beam (reference beam) of intensity $I_p$. The two-beams being coherent, interfere inside the crystal and produce an interference pattern. That is creating the necessary hologram. The image of the region of interest inside the crystal is extremely small, essentially a highly localized dynamic grating is formed. The c-axis of the crystal is oriented with respect to the optical axis in such a way that the object beam gets amplified at the expense of the pump beam [8]. Since the BaTiO$_3$ crystal is having a slow response time, hence the two-beam coupling process takes a finite time to reach the steady state. At this instant of time at the observation plane, we obtain an enhanced intensity along the direction of the object beam due to transfer of energy from the strong pump beam, $I_p$ to the weak scattered object beam, $I_o$. The parameters that determine the efficiency of the energy transfer depends on (i) the angle between the interfering beams, (ii) the angle between the index grating and the c-axis of the crystal and (iii) the modulation ratio of the interfering beams [8].

After allowing sufficient time for the two-beam coupling process to reach steady state, an arbitrary phase shift $\alpha$ is introduced by varying the pressure inside a quartz cell inserted in the pump beam. Phase shift $\alpha$ introduces a constant phase variation in the object beam and this results in a disturbance of two-beam coupling process. At this instant of time at the observation plane (GG), we have two beams;

i) the first one constitutes the initial state of the object, that is, due to the diffraction of the pump beam from the grating formed at the crystal plane

and

ii) the second scattered beam that represents the directly transmitted object beam which constitute the present phase shifted state of the object with unknown phase shift $\alpha$.
In other words, a speckle field generated from the decaying dynamic grating and a phase shifted speckle field directly transmitted from the object interacts at the observation plane. The amplitudes of the two interacting speckle fields at some instant of time \( t \) can be expressed as:

\[
A_1(t) = A_{01}(t) \exp(i\phi) \\
A_2(t) = A_{02}(t) \exp(i\phi + \alpha)
\]  

(1)  

(2)

where \( \phi \) is the random phase of the speckle field; \( A_{01}(t) \) and \( A_{02}(t) \) are the amplitudes of the initial and the phase shifted speckle fields at the observation plane.

In the two beam coupling process, the amplitudes of the interacting speckle fields vary as a function of time. The amplitude \( (A_{01}) \) of the speckle field diffracted from the grating (initial state) keeps decreasing due to the continuous read out of the dynamic grating by the pump beam, whereas the amplitude \( (A_{02}) \) of the new phase shifted speckle field increases.

At a given instant of time during the two-beam coupling process, the amplitudes, \( A_{01} = A_{02} = A_0 \), and hence, the total intensity distribution at the observation plane can be expressed as:

\[
I = |A_0 \exp(i\phi) + A_0 \exp(i\phi + \alpha)|^2 = 2I_0(1 + \cos \alpha)
\]  

(3)

where \( I_0 = |A_0|^2 = |A_{01}|^2 = |A_{02}|^2 \);

A theoretical intensity variation profile as a function of phase term \( \alpha \) is shown in Fig. 2

(a) case I: phase shift \( \alpha = 0 \)

In the case of zero phase shift, Equation (3) reduces to
\[ I = 4I_0 \]  \hspace{1cm} (4)

The total intensity \( I \) in Eq.(4) represents the steady state two-beam coupling transfer of energy from the pump beam to the object beam.

\( (b) \quad \text{case II: Phase shift} \alpha = \pi \)

Varying the pressure inside the quartz cell results in path length change in the set-up and when the phase shift \( \alpha = \pi \), then the Equation (3) reduces to zero. Destructive interference creates between the two speckle patterns at the observation plane; that is, newly transmitted speckle field from the object is completely phase or contrast reversed with respect to the initial speckle field which is formed due to the diffracted pump beam from the grating. This condition repeats when ever the phase shift \( \alpha = (2n+1)\pi; \ n=0,1,2,3... \) The destructive intensity plot is also represented in Fig.2.

\( (c) \quad \text{case III: Phase shift} \alpha = 2\pi \)

For the phase shift value of \( \alpha=2\pi \), the intensity distribution is same as given by Equation (4) and the intensity distribution follows the steady-state intensity provided that the observations are made with in the decay time of the dynamic grating created inside the crystal. As shown in Fig.2, the condition occurs when ever the phase shift \( \alpha \) is even multiples of \( \pi \), (\( \alpha=2n\pi \)).

3.2 Experimental Results

The experimental investigation to analyze the speckle patterns because of implementation of phase reversal technique in a two-beam coupling configuration is carried out on a thin ground glass diffuser mounted on a rotational stage. The object is scattered after passing through the ground glass and the resulting speckle pattern is imaged by the imaging lens (f=90 mm) onto the BaTiO\(_3\) crystal. The magnification of the imaging system is 0.65. The pump beam derived from the same laser source is added at an angle 10° (external) with respect to the object beam axis. The optical set-up consists of
a fabricated quartz cell (QC) (50x50x5 mm\(^3\)), pressure pump (PP) and a precision pressure gauge (PG) of measuring range from 0-6.0 kg/cm\(^2\) with a sensitivity of \(\pm 0.005\) kg/cm\(^2\). The quartz cell is inserted in the pump beam. The pump beam interferes with the object beam inside the crystal and creates the necessary grating.

With the present experimental parameters, we have measured the rise and eraser rates, and also the diffraction efficiency as a function of time by monitoring the object beam intensity with time using Newport power meter (Model 835) placed at the observation plane. Initially the pump beam (\(I_p\)) is cut-off, hence the intensity that reaches the power meter is mainly due to the direct speckle field generated from the diffuser. At a time \(t\), the pump beam is allowed to interfere with the imaged speckle field at the crystal. The intensity of the scattered object speckle field grows with time. Once the two-beam coupling is established, the object beam intensity starts increasing and the power meter reading is continuously gathered with time till the two-beam coupling process reaches the steady state (saturation state). At this instant time grating has reached its maximum index modulation. On blocking the object beam, only the diffracted speckle field generated by the pump beam from the dynamic grating reaches the power meter. Since the read out process is destructive, the diffracted beam intensity decreases with time. The growth of the intensity of the transmitted speckle field due to the two-beam coupling, and also the decay of the diffracted speckle field when the speckle field from the object is blocked, are plotted and shown in Fig.3. The corresponding diffraction efficiency calculated at various intervals of time is shown Fig.4

Fig.5(a) shows the photograph of the speckle pattern when the two beam coupling process reaches steady state. Now the pressure inside the quartz cell is varied in a controlled manner, and the intensity of the speckle fields at the observation plane are measured. At a particular pressure, the intensity reduces to minimum and this applied pressure corresponds to a phase shift of \(\pi\). Fig.5(b) shows the corresponding photograph of a phase reversal speckle pattern. The intensity variation with respect to the applied pressure inside the cell is also shown in Fig.5(c). In the present analysis, only small pressures are needed for obtaining the required phase shifts, and the phase step is linear with reference to the applied pressure as shown in Fig.6. The experimental results show that a 0.2 kg/cm\(^2\) input pressure provides a phase shift of \(\pi\) to achieve a phase reversed speckle pattern.
4. PHASE SHIFTING SPECKLE PHOTOGRAPHY

4.1 Theory

In the section 3, we have analyzed the influence of phase shift variation on speckle pattern. In addition to the phase shift, if the object is also simultaneously subjected to load in a two-beam coupling configuration; at the observation plane, we have two speckle fields;

i) one that represents the diffracted pump beam from the dynamic grating created in the initial stage

and

ii) the second speckle field that is directly transmitted deformed phase shifted speckle field through the crystal.

The overlap of these two identical but displaced speckles gives rise to a fringe pattern. The additional phase shift $\alpha$ modifies the position of the bright and dark fringes within the fringe system. The intensity distribution of fringe pattern at the observation plane can be written as

$$I = 2I_0 [1 + V \cos(\Omega + \alpha)]$$  \hspace{1cm} (5)

where $V$ is the fringe visibility and $\Omega$ is the phase change due to the object deformation. The phase change $\Omega$ is responsible for positional shift of the speckles at the observation plane.

The fringe visibility $V$ is defined in terms of the diffracted pump beam intensity $I_{01} = |A_{01}|^2$ and the directly transmitted object beam intensity $I_{02} = |A_{02}|^2$ as

$$V = \frac{2\sqrt{I_{01}I_{02}}}{I_{01} + I_{02}}$$  \hspace{1cm} (6)
As stated earlier that the intensity of the diffracted beam keeps decreasing due to the continuous read out of the grating by the pump beam and the intensity of the new object beam increases because of the two-beam coupling. Since the change in the intensity of the interfering beams varies with time, the visibility of the fringes, $V$ obtained as result of the interference of these two beams is also a function of time. The visibility of the fringes will be maximum ($\approx 1$), when these intensities at some instant of time are equal (Fig.3).

There are three unknown in a recorded intensity ($I_o, V, \Omega$), (Equation (5)). The following four frame phase shifting technique provides the fundamental equations for calculating the phase distribution $\Omega$ [13,14].

\[
\begin{align*}
I_1 & = I_o \left[ 1 + V \cos(\Omega) \right] \\
I_2 & = I_o \left[ 1 + V \cos(\Omega + \pi/2) \right] \\
I_3 & = I_o \left[ 1 + V \cos(\Omega + \pi) \right] \\
I_4 & = I_o \left[ 1 + V \cos(\Omega + 3\pi/2) \right]
\end{align*}
\]

(7)

The $\frac{\pi}{2}, \pi, \frac{3\pi}{2}$ phase shifts can be provided from the calibrated phase shifter discussed in Section 3.

The phase distribution $\Omega$ that is responsible for the position shifted of the speckles can be obtained from the following arctan function

\[
\Omega = \arctan \left[ \frac{I_4 - I_2}{I_1 - I_3} \right]
\]

(8)

If $\beta$ is the angle between the zero order and $n^{th}$ order fringe at the lens $L_2$; the angular separation of the fringe pattern satisfies the relation [1]

\[
d_0 = \frac{n\lambda}{M \sin \beta}
\]

(9)
From the evaluated phase, the surface displacement, \( d_0 \), can be obtained as

\[
d_0 = \frac{\Omega \lambda}{2\pi M \sin \beta}
\]  

(10)

### 4.2 Experimental Analysis

The experimental arrangement is same as described in Section 3.1. Once the two-beam coupling process reaches its steady state, the object is given a small in-plane rotation. The real-time speckle fringe pattern that represents the condition for zero phase shift, seen at the observation plane, is recorded as shown in Fig.7(a). The pressure inside the quartz cell is varied in a controlled manner and the fringe patterns corresponding to phase shift values of \( \pi/2, \pi, 3\pi/2 \) are also recorded (Fig.7(b)-7(d)). A discrete step method is employed for recording the images; the time delay between the consecutive steps is much longer than that required for establishing two-beam coupling [7,12]. In this method the writing beams are allowed to fall on the crystal only during the recording of each phase-shifted fringe pattern. These four stored fringe patterns are used for evaluating the phase \( \Omega \) from the equation (8). The patterns are low pass filtered with a 3 x 3 window, and the resulting phase map and the evaluated displacement are shown in Fig.8 and Fig.9 respectively.

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### 5. CONCLUSION

A real-time photorefractive phase reversal speckle photography system is developed to investigate the influence of \( \pi \)-phase shift on the random speckle pattern and subsequently on the fringe pattern. It is shown that phase reversal is achieved when a \( \pi \) - phase shifted speckle pattern overlaps on a un-shifted speckle pattern at the observation plane. The present analysis paves a way for implementation of a real-time approach for phase shift calibration in speckle photography. Further, it is also focused point beam analysis of speckle correlation Young’s fringes by implementing the phase shifting technique. These results illustrate not only that photorefractive crystals are a suitable recording material for real-time visualization of fringe pattern in speckle photography, but also that their slow response time can be exploited to store the phase-shifted correlation speckle fringe patterns for evaluation, by skillful management of the exposure time.
LIST OF FIGURE CAPTIONS

Figure 1
Schematic of a phase reversal speckle photography (PRSP) arrangement
VBS – Variable Beam Splitter, DG – Diffuser, QC – Quartz Cell, PP – Pressure Pump,
PG – Pressure Gauge, GG – Ground Glass, $I_o$ – Object beam intensity,
$I_p$ – Pump beam intensity.

Figure 2
Theoretical intensity distribution profile.

Figure 3
Intensity plot of the object beam on the observation plane and the diffracted beam as a function of time.
The points represent the experimentally obtained intensities at various times. The growing object beam corresponds to the gain in intensity of the object resulting from two-beam coupling and the decaying beam corresponds to the erasure of dynamic grating.

Figure 4
Diffraction Efficiency as a function of time.

Figure 5
(a) Photograph of the speckle pattern when the two beam coupling process reaches steady state,
(b) Corresponding photograph of a phase (contrast) reversal speckle pattern.
(c) The intensity variation plot with respect to the applied pressure inside the quartz cell.

Figure 6
Experimentally calibrated phase shift plot as a function of applied pressure.

Figure 7
In-plane displacement fringe patterns when the phase shift (a) $\alpha=0$, (b) $\alpha=\pi/2$, (c) $\alpha=\pi$ and (d) $\alpha=3\pi/2$

Figure 8
Phase map computed from the recorded fringe patterns using four-bucket algorithm.

Figure 9
3-D Plot of computed phase
References

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VBS – Variable Beam Splitter, DG – Diffuser, QC – Quartz Cell, PP – Pressure Pump,
PG – Pressure Gauge, GG – Ground Glass, $I_o$– Object beam intensity,
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Fig. 2 Theoretical intensity distribution profile.
**Fig.3** Intensity plot of the object beam on the observation plane and the diffracted beam as a function of time. The points represent the experimentally obtained intensities at various times. The growing object beam corresponds to the gain in intensity of the object resulting from two-beam coupling and the decaying beam corresponds to the erasure of dynamic grating.

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Fig. 4 Diffraction Efficiency as a function of time.
**Fig. 5** (a) Photograph of the speckle pattern when the two beam coupling process reaches steady state, (b) Corresponding photograph of a phase (contrast) reversal speckle pattern. (c) The intensity variation plot with respect to the applied pressure inside the quartz cell.
Fig. 6 Experimentally calibrated phase shift plot as a function of applied pressure.
Fig. 7 In-plane displacement fringe patterns when the phase shift (a) $\alpha=0$, (b) $\alpha=\pi/2$, (c) $\alpha=\pi$ and (d) $\alpha=3\pi/2$
Fig. 8  Phase map computed from the recorded fringe patterns using four-bucket algorithm.
Fig. 9 3-D Plot of computed phase