Synopsis of

PHOTONIC BAND GAP STUDIES AT MICROWAVE FREQUENCIES USING DIELECTRIC AND MAGNETIC MATERIALS

A THESIS

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DURGA VENKATA NAGESH EVANI

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DEPARTMENT OF PHYSICS
INDIAN INSTITUTE OF TECHNOLOGY MADRAS
CHENNAI 600 036, INDIA

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1. INTRODUCTION

The electromagnetic wave propagation in periodic structures has been extensively studied (Meade et al, 1991, 1992, Zhang and Satpathy, 1990, Romo and Smy, 2003) in the past two decades due to wide range of applications such as filters, switches, antennas, resonators etc. The structures constructed in microwave region could be used in antennas (Brown and MacMahon, 1996) and the structures constructed in optical region can be used for the suppression of spontaneous emission in lasers (Yablonovitch 1987). These structures prepared with dielectric or magnetic materials, do not allow electromagnetic waves of certain frequencies. This phenomenon is similar to the electronic band gaps in crystals. Hence, these structures are called Photonic Band Gap (PBG) structures or Photonic Crystals (PCs). As there is a similarity between real crystals and these structures, the nomenclature of real crystals can also be carried over to the electromagnetic case. Therefore, the concepts of reciprocal space, Brillouin zones, dispersion relations, Bloch wave functions etc. can also be applied to photonic case (Yablonovitch, 1993). But the Brillouin zone of PCs is approximately 1000 times smaller than that of the electronic case.

A PBG is specifically defined as a range of frequencies over which the propagation of electromagnetic waves is forbidden. The formation of the band gap can be explained basically by two reasons viz., (1) the dielectric and/or magnetic potential created by the sample rods and (2) the Bragg diffraction. Though we can have analogy with electronic band gaps, the dispersion relations are completely different. These structures can be constructed over a wide frequency region specifically from microwave to visible region. In acoustic region also, these structures can be prepared and are called Phononic Band Gaps (Khelif et al, 2003). Photonic crystals can also be made by drilling holes in a high dielectric constant material. Also, depending on the
polarization of the incident electric and magnetic fields, one can study these crystals with E –
polarization or H – polarization.
As mentioned earlier, the dielectric material that is used in constructing a photonic crystal acts as
a potential. So, as long as the dielectric constant of the material is greater than unity, one can
expect a band gap. But in general, for a complete band gap to appear there will be a minimum
dielectric constant that varies with the geometry of the structure under consideration. However,
one can always get a band gap in the normal direction even for low values of dielectric constant.
Also, in certain applications such as antennas the normal direction is important and maximum
intensity can be achieved in this direction (Brown and MacMahon, 1996). Hickmann et al
(Hickmann et al 2002) constructed photonic band gap structures by cementing acrylic rods
$(\varepsilon'=2.58)$ in a hexagonal array to form rectangular stacks.
Most of the applications of photonic crystals depend on two main factors; due to efficient
reflectors within the band gap frequencies and the creation of defect mode within the band gap.
By creating a defect, the structure acts as a resonator at the defect site resulting in a defect mode.
The width of the defect mode tends to zero as the size of the crystal tends to infinity
(Joannopoulos et al, 1995). The defects can be either point defects or line defects. Defects can be
created in different ways such as removing a sample from a particular position, insert a sample at
a particular interstitial site, increasing the refractive index locally and by changing the radius of
the sample locally. By creating the line defects, the structure can be made to act like a Fabry-
Perot resonator or waveguide depending on the direction of propagation (Temelkuran and
Ozbay, 1999). Fabry-Perot resonators can be used for tuning mechanism or for a MASER cavity
(Serpenguzel, 2002).
The photonic band gap structures can be analyzed theoretically using various methods such as plane wave method (Plihal and Maradudin, 1991), transfer matrix method (Sigalas et al, 1994), finite difference time domain method (Yamada et al, 2002) and finite element method (Pendry and MacKinnon, 1992) etc. Of all these methods, plane wave method is best suited and is widely used to know the position of the band gap.

2. OBJECTIVES AND SCOPE OF THE WORK

The photonic band gap structures suitable for microwave frequency region can be easily fabricated as the length scales are in centimeters. In the microwave frequency region, the effect of permeability of the material used has to be considered while analyzing the band structure. While much of the literature concentrates on pure dielectric materials, only few reports are available on magnetic materials with no clear explanation of the band gap parameters (Hong et al 2003 and Kee et al, 1999, 2000). The appearance of the band gaps depends on various parameters such as dielectric constant, permeability, radius of the sample, lattice spacing and geometry of the structure and polarization of the electromagnetic radiation. For a given dimension, a structure having higher symmetry results in a larger gap width (Bjarklev et al, 2004). However, to the best of our knowledge, not many reports are available in the literature for the variation of the gap width and mid-gap frequency with the above parameters individually and this inspired us to study the photonic crystals using dielectric and magnetic materials in both E and H-polarizations. It is also important to analyze the structure using theoretical formulation and compare with the experimental results. In this thesis work, two dimensional microwave band gap structures are considered as they block certain frequencies of electromagnetic waves at any angle within the plane of periodicity. Thus, the main objectives of the present work are
1. To construct two-dimensional photonic band gap structures in the microwave frequency range using different dielectric and magnetic materials.

2. To develop a code using plane wave expansion method and analyze the structures theoretically and experimentally.

3. To create point defects as well as line defects in pure structures and analyze the behaviour of the defect modes.

3. SUMMARY OF RESEARCH WORK

The thesis work is summarized in three main sections viz. the theoretical simulation of the band structure for various structures, experimental observation of the transmission spectra of these structures for correlation with the theoretical prediction and experimental study on the effect of defects created in the pure structures.

3.1 Theoretical simulation: As mentioned earlier, in this thesis work, the plane wave expansion method has been chosen to analyze the structures theoretically. In this method, the fields are expanded in terms of Bloch waves as the medium is periodic. The eigen value equation for the band structure calculations is solved starting from Maxwell’s equations for E and H-polarizations.

For the dielectric case, the master equation for calculating the band gaps is given by

\[
\frac{1}{\varepsilon(x)} \nabla \times \nabla \times \vec{E}(x) = \frac{\omega^2}{c^2} \vec{E}(x)
\]  

(1)

Expanding \( \varepsilon^{-1}(\vec{x}) \) in Fourier space and expanding electric field using Bloch’s theorem one can get the eigen value equation in case of E-polarization as
\[ \sum_{\vec{G}'} \hat{K} (\vec{G} - \vec{G}') | \vec{k} + \vec{G} || \vec{k} + \vec{G}' | C (\vec{k} + \vec{G}') = \left( \frac{\omega^2}{c^2} \right) C (\vec{k} + \vec{G}). \]  

(2)

Similarly in case of H-polarization, one can get the

\[ \sum_{\vec{G}, \vec{G}'} \hat{K} (\vec{G} - \vec{G}') A (\vec{k} + \vec{G}) (\vec{k} + \vec{G}') = \frac{\omega^2}{c^2} A (\vec{k} + \vec{G}). \]  

(3)

In the case of magnetic materials, permeability also has to be considered along with the dielectric constant and the eigen value equations take the form

\[ \sum_{\vec{G}, \vec{G}'} \epsilon^{-1} (\vec{G}_0 - \vec{G}') \mu^{-1} (\vec{G}' - \vec{G}) (\vec{k} + \vec{G}) (\vec{k} + \vec{G}) b (\vec{G}) \frac{|\vec{k} + \vec{G}_0|}{|\vec{k} + \vec{G}|} = \frac{\omega^2}{c^2} b (\vec{G}_0). \]  

(4)

and

\[ \sum_{\vec{G}, \vec{G}'} \epsilon^{-1} (\vec{G}_0 - \vec{G}') \mu^{-1} (\vec{G}' - \vec{G}) b_1 (\vec{G}) (\vec{k} + \vec{G}_0) (\vec{k} + \vec{G}') = \frac{\omega^2}{c^2} b_1 (\vec{G}_0). \]  

(5)

for E and H-polarizations respectively. Here \( \vec{G} \) is a reciprocal lattice vector, \( \vec{k} \) is a wave vector, \( \omega \) is the eigen frequency, \( c \) is the velocity of light, \( \vec{E} \) and \( \vec{B} \) are the electric and magnetic fields respectively, \( \epsilon (\vec{x}) \) and \( \mu (\vec{x}) \) are the periodic functions of dielectric constant and magnetic permeability and \( \hat{K} (\vec{G}) \) is the Fourier coefficient.

The above equations are the eigen value equations and a plot of the dispersion relation gives an idea about the position of the band gap. A code has been developed to obtain the band structures for E and H polarizations in the irreducible Brillouin zone. Figs. 1a and 1b show typical square and triangular lattices as viewed from the top and the corresponding Brillouin zones are shown in figs 1c and 1d. A typical band structure for a square lattice of material of dielectric constant 5.5 is as shown in figure 2.
Fig. 1: a) Square lattice as viewed from the top b) Triangular lattice as viewed from the top c) The Brillouin zone for square lattice and d) The Brillouin zone for triangular lattice. The triangular wedge $\Gamma$-X-M is irreducible Brillouin zone in case of square lattice and $\Gamma$-M-K in case of triangular lattice.

Fig. 2: A typical band structure shown for a square lattice of material of dielectric constant 5.5.
The band gap parameters such as gap width, mid-gap frequency are studied extensively in both square and triangular lattices with both E and H-polarizations theoretically in dielectric as well as the magnetic case. The variation in the gap width and mid-gap frequency is also studied in the dielectric as well as the magnetic case for the structures in which a low dielectric material occupies the lattice sites in a high dielectric material (i.e making holes in a high dielectric material).

3.2 Construction of microwave band gap structures
Low dielectric constant materials such as Poly Tetra Fluoro Ethylene (PTFE, $\varepsilon'$=2.10), Poly Vinyl Chloride (PVC, $\varepsilon'$=2.38 and $\varepsilon''=0.12$) and a relatively high dielectric constant material, glass ($\varepsilon'$=5.50 and $\varepsilon''=0.1$) are used in case of dielectric materials, and lithium zinc ferrite ($\varepsilon'$=9.87, $\varepsilon''=0.14$, $\mu'$=2.17 and $\mu''=0.12$) is chosen as a magnetic material. The dielectric permittivity and magnetic permeability of these materials are measured at 10 GHz using cavity perturbation technique. In order to construct a microwave band gap structure, it is better to select the proper lattice spacing at which the width of the band gap is maximum. Using plane wave expansion method, the lattice spacing for maximum gap width (or gap to mid-gap ratio) is chosen to construct the structures in all the cases. For PTFE rods, the chosen lattice spacing is 1.0 cm for both square and triangular lattices. For PVC rods, the lattice spacings are 1.0 cm for square lattice and 1.1 cm for triangular lattice. In case of glass samples, the spacing is 0.9 cm for both square and triangular lattices. The background dielectric medium is chosen as air and the samples are inserted in a thermocol sheet (whose dielectric constant is close to that of air) in square and triangular geometries for experimental purpose.
3.2.1 Preparation of Lithium Zinc Ferrite:

Conventional solid state reaction method is followed in preparing the compound. The calcination and sintering temperatures are maintained at 800 and 950°C respectively. The single phase formation is confirmed by XRD. The ferrite rods of approximately 10 cm length are prepared using extrusion technique.

3.2.2 Study of band gaps:

The microwave band gap structures are characterized by two main parameters viz., gap width and mid-gap frequency. Figure 3 shows the transmission spectra obtained for a glass structure arranged in a square lattice with and without defects and the band structure for the structure without defects in the normal direction (Γ-X direction). The radius of the rods is 0.207 cm and the lattice spacing is 0.9 cm. Table 1 shows the gap width, mid-gap frequency and gap to mid-gap ratio obtained theoretically and experimentally for three dielectric materials.

![Fig. 3: The transmission spectrum and the band structure in the normal direction.](image)
Table 1. The theoretical and experimental values of gap width and mid-gap frequency obtained for square and triangular lattice structures of different materials.

<table>
<thead>
<tr>
<th>Sample</th>
<th>Lattice</th>
<th>Theoretical</th>
<th>Experimental</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Mid-gap (GHz)</td>
<td>Gap width (GHz)</td>
</tr>
<tr>
<td>PTFE</td>
<td>Square</td>
<td>13.14</td>
<td>2.04</td>
</tr>
<tr>
<td></td>
<td>Triangular</td>
<td>14.84</td>
<td>2.20</td>
</tr>
<tr>
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<td>2.32</td>
</tr>
<tr>
<td></td>
<td>Triangular</td>
<td>13.18</td>
<td>2.31</td>
</tr>
<tr>
<td>Glass</td>
<td>Square</td>
<td>12.87</td>
<td>4.64</td>
</tr>
<tr>
<td></td>
<td>Triangular</td>
<td>14.18</td>
<td>5.15</td>
</tr>
</tbody>
</table>

3.2.3 Study of band gaps in magnetic materials:

As the gap width and mid-gap frequency in general depend on the refractive index of the material, use of magnetic material could be useful in improving the gap parameters. By including the permeability, the refractive index $\sqrt{\mu\varepsilon}$ increases indicating a possibility of attaining wider band gaps (Sigalas et al, 1997). On the contrary it is found that the gap width decreases with increase in permeability. The mid-gap frequency is observed to decrease as $\mu$ increases. Fig. 4 shows the variation in the gap width and mid-gap frequency with permeability obtained numerically for a square lattice of lattice spacing 0.7 cm and radius as 0.16 cm (E-polarization).
Fig. 4: The variation in the gap width and mid-gap frequency with permeability.

A similar behaviour is observed in case of H-polarization also. Comparing with the dielectric case, the mid-gap frequency shows a similar behaviour whereas the gap width increases with increase in dielectric constant. Therefore, it is proposed in this thesis work that both the refractive index $\sqrt{\mu \varepsilon}$ and wave impedance $\sqrt{\frac{\mu}{\varepsilon}}$ are responsible for the variation in the gap parameters.

3.3 Effect of defects: Defects can be created in pure structures and they can be of two types (Johnson et al, 2004)

1. Point defects: Defect modes are found to appear in square and triangular lattices with the creation of point defects in pure structures. Fig. 3 shows the defect mode in case of 0.9 cm square lattice. It is found that not all defect sites give rise to defect modes and the appearance of the defect mode is position dependent. Also, no defect mode is observed in case of structures made of PVC and PTFE.
2. Line defects: The defect mode obtained from Fabry-Perot resonators constructed by creating line defects in all the cases is found to shift towards lower frequency side as the defect spacing is increased indicating the property of a resonator. Also, the Q-factor of the defect mode is dependent on dielectric constant and is found to increase with increase in dielectric constant.

4. Conclusions

The major conclusions derived from the present study are as follows:

- Microwave band gap structures can be constructed even with low dielectric material and an appreciable value of gap width can be obtained by properly choosing the lattice spacing.
- The gap width in case of triangular lattice of air holes is more when compared to that of rods in a low dielectric background.
- For finite sized structures, the appearance of the defect modes in case of point defects is found to be position dependent.
- The sharpness (Q factor) of the defect mode depends on the localization of the electromagnetic waves at the defect site. It is concluded that high dielectric contrast (ratio of dielectric constant of the medium to that of the background) results in higher localization resulting in a high Q-factor defect mode.
- It is important to consider both refractive index and impedance of the material to analyze the gap width and mid-gap frequency.
REFERENCES


**Proposed Plan of Thesis Chapters**

**Chapter 1:** Introduction

**Chapter 2:** Plane wave method for photonic band gap structures.

**Chapter 3:** Experimental details

**Chapter 4:** Study of band gap structures with dielectric materials in square and triangular lattices and the effect of point defects.

**Chapter 5:** Effect of defect spacing in Fabry-Perot resonators constructed by creating line defects.

**Chapter 6:** Effect of magnetic permeability on the band gap parameters

**Chapter 7:** Summary and Conclusions
LIST OF PUBLICATIONS BASED ON THE RESEARCH WORK

The main results of the present work have been published / accepted as given below.

Publications in refereed journals


Accepted:

1. E. D. V. Nagesh, V. Subramanian, V. Sivasubramanian and V. R. K. Murthy Numerical study of the effect of permeability on square and triangular microwave band gap structures, Accepted for publication in *Physica B: Condensed matter*

Presentations in Conferences


