Current status of inflationary cosmology

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Recent observations have determined basic cosmological parameters in high precisions.

However it also shows how we do not understand the universe!
Three unsolved problems

1. What is the origin of dark matter?
   Neutrinos?
   Super-symmetric particles?

2. What is the origin of dark energy?
   Cosmo-illogical constant?
   French wine?
   Modified gravity?

3. What is the origin of inflation?
   Inflation really occurred in early universe?
   Inflation is driven by a scalar field or by some other mechanism?
**Inflation**

From Einstein equations, the scale factor satisfies

\[
\frac{\ddot{a}}{a} = -\frac{4pG}{3}(1 + 3w)\rho \quad \text{where} \quad w = \frac{p}{\rho}
\]

If \( w < -\frac{1}{3} \), we have an accelerated expansion

\[\ddot{a} > 0\]

The amount of inflation is quantified by the number of e-folds:

\[N = \log \left( \frac{a}{a_i} \right)\]

We require \( N > 70 \) to solve flatness and horizon problems.

10^{-30} \text{ cm} \rightarrow e^{70} \text{ times} \rightarrow 1 \text{ cm} \quad \text{(just after the end of inflation)}
The idea of inflation was proposed by several physicists independently:


The abstract of the Kazanas’s paper:

"... The expansion law of the universe then differs substantially from the relation considered so far for the very early time expansion. In particular it is shown that under certain conditions this expansion law is exponential. It is further argued that under reasonable assumptions for the mass of the associated Higgs boson this expansion stage could last long enough to potentially account for the observed isotropy of the universe."

He mentioned that inflation can solve a flatness problem!
Starobinsky’s model (1980)

Around the Planck scale, the effect of higher-order curvature terms can be important.

\[ L = R + \alpha R^2 \]

The \( R^2 \) term leads to an exponential expansion. Inflation ends after the \( R^2 \) term becomes unimportant relative to \( R \).

\[ G_{\mu\nu} = 8\pi G T_{\mu\nu} \]

The modified gravity model has been also in active debate in the context of dark energy.

Changing gravity
Inflation based on GUTs (using a scalar field)

Kazanas

Sorry not having a photo!

Sato

Guth

Inflation occurs because of first-order phase transition of a vacuum.

First-order phase transition of a vacuum and the expansion of the Universe

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Inflationary universe: A possible solution to the horizon and flatness problems

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So far many inflation models have been proposed. Most of them make use of scalar field.

\[ G_{\mu\nu} = 8pGT_{\mu\nu} \]

Introducing a scalar field: \( f \)

Two many models!

old, new, chaotic, extended, power-law, hybrid, natural, supernatural, extra-natural, eternal, D-term, F-term, P-term, winter-term, brane, D-brane, oscillating, tachyon, dilaton, modulus, string-(un)inspired, ghost condensate, \( \ldots \) (perhaps more than 100 models!)
Linde wrote 122 papers whose titles include the words ‘inflation’ or ‘inflationary’.

- New Inflation (1981, 1624 citations)
- Chaotic Inflation (1983, 998 citations)
- Hybrid Inflation (1994, 529 citations)
- KKLMMT inflation (2003, 347 citations)

But Andrei, which models are favoured?
Model buildings of inflation are important.

But at the same time we need to find ways to discriminate between a host of inflation models!

Observations can tell us something on model discriminations?
In order to confront inflation observations, we need to study density perturbations generated during inflation.

Before doing that let’s consider background dynamics during inflation.

See the reviews:

Linde, hep-th/0503203
Liddle and Lyth, ‘Cosmological inflation and Large-scale structure’
Lyth and Riotto, hep-th/9807208
Bassett, Tsujikawa and Wands, astro-ph/0507632
Standard Inflation scenario

\[
S = \int d^4x \left[ \frac{R}{2} - \left( \frac{\ddot{\phi}}{2} \right)^2 / 2 - V(f) \right]
\]

\[H^2 = \frac{8\pi G}{3} \left( \frac{1}{2} \dot{\phi}^2 + V(f) \right) \gg \frac{8\pi G}{3} V(f)\]

\[H \gg \text{const} \quad \Rightarrow \quad a \propto e^{Ht}\]

\(f:\) inflaton

Minimally coupled to gravity
During inflation, the comoving Hubble radius

\[ H^{-1}/a \]

decreases.

This provides a causal mechanism to generate density perturbations.
Perturbations in standard slow-roll inflation

Inflaton: \( f = f_0 + df \)

Metric :
\[
ds^2 = -(1 + 2A)dt^2 + 2a \Phi_i Bdx^i dt + a^2 [(1 - 2y) d_{ij} + 2 \Psi_{ij} E + h_{ij}] dx^i dx^j
\]

Scalar perturbations: \( A, B, E, y \)

Tensor perturbations: \( h_{ij} \)

Several gauge invariant quantities are constructed.

Comoving curvature perturbations: \( z = y + \frac{H}{f} df \)
Equation of scalar perturbations

Perturbed Einstein equations

\[ dG_{\mu\nu} = 8 \rho dT_{\mu\nu} \]

give

\[ \frac{1}{az^2} \frac{d}{dt} \left( az^2 \dot{z} \right) + \frac{k^2}{a^2} z = 0 \]

where \[ z^2 = \frac{a^2 \dot{f}^2}{H^2} \]

k is a comoving wavenumber.

In the large-scale limit we have

\[ z = C_1 + C_2 \int \frac{dt}{az^2} \]

\[ \Rightarrow z = \text{const} \]

Perturbations are ‘frozen’ for \[ 2\rho/k >> H^{-1}/a \] i.e. \[ k << aH \].

Decaying mode
Spectra of scalar perturbations

Using a slow-roll analysis, we get

\[ z \gg H \frac{df}{f} \quad \text{and} \quad P_s \gg \frac{V^3}{(M_p^6 V, f^2)} \]

\[ df \@ H/2p \]

(quantum fluctuations)

The spectral index is

\[ n_s \gg 1 - 6e + 2h \]

\[ e = \frac{1}{2k^2} \left( \frac{V, f}{V} \right)^2 \quad \text{and} \quad h = \frac{V, ff}{k^2 V} \]

Since \( \epsilon, \eta \ll 1 \), we get a nearly scale invariant spectrum with

\[ n_s \gg 1 \quad \text{(general inflationary prediction)} \]
Tensor perturbations

The equation for tensor perturbations is

\[ \ddot{h} + 3H \dot{h} + \frac{k^2}{a^2} h = 0 \]

The tensor perturbations generated in inflation is

\[ h \gg H/(2pM_p) \]

The spectrum and the spectrum index are

\[ P_T \gg V / M_p^4 \mu k^{n_T} \]

Tensor-to-scalar ratio:

\[ r = P_T / P_S = 16 e \]

Consistency relation:

\[ n_T = -r / 8 \]
Observables

We have six inflationary observables:

\( P_S \): Scalar amplitude
\( r \): Tensor-to-scalar ratio
\( n_S \): Spectral index of scalar perturbations
\( n_T \): Spectral index of tensor perturbations
\( \alpha_S \): Running of scalar perturbations
\( \alpha_T \): Running of tensor perturbations

\( n_T \) and \( r \) are related through the consistency relation:

\[ n_T = -\frac{r}{8} \]
Slow-roll analysis

Using slow-roll parameters

\[
e = \frac{1}{2k^2} \left( \frac{V_{,f}}{V} \right)^2
\]

\[
h = \frac{V_{,ff}}{k^2 V}
\]

\[
x^2 = \frac{V_{,f} V_{,fff}}{k^4 V^2}
\]

we have

\[
r = 16e
\]

\[
n_S = 1 - 6e + 2h
\]

\[
n_T = -2e
\]

\[
\alpha_S = 16eh - 24e^2 - 2x^2
\]

\[
\alpha_T = -4e(2e - h)
\]

Three independent quantities \( e, h, x \)
and the amplitude \( P_S \)
Classification of inflation models

(I) Large-field

\[ V(f) = V_0 f^p \]

(II) Small-field

\[ V(f) = V_0 \left[ 1 - \left(\frac{f}{m}\right)^p \right] \]

(III) Hybrid

\[ V(f) = V_0 \left[ 1 + \left(\frac{f}{m}\right)^p \right] \]
Classification in the \((n_s, r)\) plane

\[ h = e \]
(characterized by an exponential potential)

\[ h = -e \]
(characterized by a linear potential)

Hybrid: \(0 < \epsilon < \eta\)

Large Field: \(-\epsilon < \eta < \epsilon\)

Small Field: \(\eta < -\epsilon\)
Observational constraints from WMAP 3-yr data

Harrison-Zeldovich spectrum is under observational pressure.
Some models can be ruled out?

In the large-field model with $V = V_0 \phi^p$,

$$n_s - 1 = -\frac{2(2p+1)}{N(p+2)}$$

$$r = \frac{24p}{N(p+2)}$$

For fixed $p$, $n_s$ and $r$ are only dependent on the e-folds $N$.

$$N = \ln \left( \frac{a_f}{a} \right)$$

When $p=2$ and $N=55$, $n_s = 0.964$ and $r=0.144$.

When $p=4$ and $N=55$, $n_s = 0.947$ and $r=0.283$. 
(A) Quartic model (p=4)
N=64 is marginal.
N<64 is ruled out at the 95% CL.

(B) Quadratic model (p=2)
N=50-60
Allowed.
How about other inflationary models?

Small-field models:  \[ V = V_0 \left[ 1 - \left( \frac{f}{m} \right)^p \right] \]

Two parameters: \( V_0 \) and \( \mu \)

Additional freedom to satisfy observational constraints

E.g., Natural Inflation: Freese et al. (1990)

\[ V(\phi) = m^4 \left[ 1 + \cos \left( \frac{\phi}{f} \right) \right] \]

Consistent with WMAP3 data for

\[ f > 0.7m_{pl} \]  \quad (Savage et al, 2006)

\[ m \sim m_{GUT} \]
Hybrid models

Linde (1994)

\[ V = \frac{\lambda}{4} \left( \chi^2 - \frac{M^2}{\lambda} \right)^2 + \frac{1}{2} g^2 \phi^2 \chi^2 + \frac{1}{2} m^2 \phi^2 \]

\( c \gg 0 \)

\[ V \approx \frac{M^4}{4 \lambda} + \frac{1}{2} m^2 \phi^2 \]

Inflation ends for

\[ \phi < \phi_c \equiv \frac{M}{g} \]

We generally have a blue-tiled spectrum with a negligible tensor-to-scalar ratio.

\[ n_s \gg 1 + \frac{\lambda m^2}{pM^4} \]

\[ r \ll 1 \]

Under observational pressure
In multi-field models, there is another possibility: **Double inflation**

In this case we have to take into account **Isocurvature perturbations**

\[ S_{fc} = \frac{df}{\dot{f}} - \frac{dc}{\dot{c}} \]  
**relative entropy mode**

See the review of Kodama and Sasaki (1984).

**Adiabatic and isocurvature perturbations are generally correlated.**
Correlated adiabatic and isocurvature perturbations

Langlois (1999), Gordon et al. (2001)

Correlation ratio:

Modified consistency relation: \[ r = -8n_T(1 - r_c^2) \]


In supersymmetric models with \( g^2/\lambda = 2 \), the correlation is actually strong:

\[ r_c \gg 1 \]  

(S.T., Parkinson and Bassett, 2003)
Constraints from WMAP

\[ V = \frac{\lambda}{4} \left( \chi^2 - \frac{M^2}{\lambda} \right)^2 + \frac{1}{2} g^2 \phi^2 \chi^2 + \frac{1}{2} m^2 \phi^2 \]

with \( g^2/\lambda = 2 \)

Parkinson, S.T., Bassett, Amendola (2005)

Likelihood values are

\[ \lambda \gg (M/m_p)^2, M/m_p \gg 5 \times 10^{-8}, m/M \gg 0.5 \]

Satisfying the condition for double inflation
Isocurvature contributions

We find the observational constraints:

\[ \frac{P_S}{P_R} < 0.004, \quad \frac{P_C}{P_R} < 0.07 \quad (2\sigma) \]

Isocurvature modes need to be suppressed.
Likelihood values of \( N_{2\text{nd}} \) are around \( 52 < N_{2\text{nd}} < 59 \).

The modes on cosmologically relevant scales are generated during the second stage of inflation (red-tilted spectrum).
Other observational constraints on inflation?

Non-gaussianities

\[ F(x) = F_G(x) + f_{NL} F_G^2(x) \]

WMAP3 yr constraints

\[ -54 < f_{nl} < 114 \]

In single-field inflation

\[ f_{nl} \approx (n_S - 1)/4 \]

The current data do not give strong constraints.

There are some models in which non-gaussianities can be large:

Curvaton, modulated reheating, multiple fields, warm inflation,....
Alternative models to inflation?

There are some other cosmological models motivated (or at least inspired!) by string theory.

- **Pre-big-bang scenario**: Veneziano, Gasperini (1991)
  \[ S_4 = \int d^4 x \sqrt{-g_4} e^{-f} [R + (\ddot{N}f)^2 + \ldots] \]
  (super inflation for negative $t_s$)

- **Ekpyrotic/Cyclic cosmologies**: Khoury, Ovrut, Steinhardt, Turok (2001)

Both scenarios make use of bouncing cosmological solutions (in Einstein frame).
Ekpyrotic universe

1. Bulk brane moves slowly moves across the bulk.
2. Bulk brane collides with our visible brane.
3. Radiation is produced around the collision.

4-dimensional effective potential

Our brane

Bulk brane

Contracting universe
Density perturbations in Ekpyrotic scenarios


The spectrum of curvature perturbations is highly blue-tilted:

\[ n_s - 1 = \frac{2}{1 - p} \]

\[ n_s @ 3 \text{ for } (\text{Ekpyrotic}) \]

\[ n_s @ 4 \text{ for } p @ 1/3 \text{ (PBB)} \]

This was also confirmed when the singularity at the bounce is avoided by including higher-order loop corrections.

S.T., Brandenberger and Finelli (2002)

Generally pressure perturbations need to be directly proportional to Bardeen potential for a pre-bounce growing mode to survive after the bounce, but this mode is not present for any known ordinary matter. (Bozza, 2006)
The reason why Ekpyrotic and PBB models do not produce scale-invariant spectra are that they are kinetically driven.

Generally models consistent with observations satisfy the requirements:

(i) The slowly varying inflaton potential

(ii) The field is not strongly coupled to gravity.

e.g., the dilaton coupling $e^{-f}$ drastically changes the spectral index.

The Starobinsky’s inflation model is exceptional, but in Einstein frame it has a slowly rolling inflaton potential.
Inflation is realized without an inflaton potential?

Let us consider the action in low-energy string theory:

$$S = \int d^4x \sqrt{-g} \left[ \frac{1}{2} F(f) R - \frac{1}{2} w(f)(\tilde{\nabla}f)^2 - \frac{1}{2} \alpha' x(f) [c_1 R_{GB}^2 + c_2 (\tilde{\nabla}f)^4] \right]$$

(i) $c_1 R_{GB}^2 \ll c_2 (\tilde{\nabla}f)^4$ ('kinetic slow-roll inflation' or 'ghost inflation')

It is possible to have inflation with $n_S \gg 1$ and $r \ll 1$. But reheating does not proceed as in the standard oscillating field.

(ii) $c_1 R_{GB}^2 \gg c_2 (\tilde{\nabla}f)^4$

Inflation is possible, but tensor perturbations show negative instability (Calcagni et al; Guo, Ohta, S.T.)

Problems of quantizations
Invalidity of linear perturbations
Summary

1. We need inflation to solve a number of cosmological puzzles.

3. Some of the models like large-field and hybrid models are under strong observational pressure.

6. Alternative 4-dimensional bouncing models like PBB and are Ekpyrotic models are in conflict with observations.

9. Slow-roll inflation models are perhaps unique models consistent with observations.

But we do not know which are the best inflation model.

Let’s see what happens in future observations and in future development of string theory!