Gaussian beam (from Wikipedia)

In optics, a Gaussian beam is a beam of electromagnetic radiation whose transverse electric field and intensity distributions are well approximated by Gaussian functions. Many lasers emit beams that approximate a Gaussian profile, in which case the laser is said to be operating on the fundamental transverse mode, or "TEM₀₀ mode" of the laser's optical resonator. When refracted by a lens, a Gaussian beam is transformed into another Gaussian beam (characterized by a different set of parameters), which explains why it is a convenient, widespread model in laser optics.

The mathematical function that describes the Gaussian beam is a solution to the paraxial form of the Helmholtz equation. The solution, in the form of a Gaussian function, represents the complex amplitude of the electric field, which propagates along with the corresponding magnetic field as an electromagnetic wave in the beam.

Mathematical form

For a Gaussian beam, the complex electric field amplitude is given by

\[
E(r, z) = E₀ \frac{w₀}{w(z)} \exp \left( -\frac{r^2}{w^2(z)} \right) \exp \left( -ikz - ik\frac{r^2}{2R(z)} + i\zeta(z) \right),
\]

where
The functions \( w(z), R(z), \) and \( \zeta(z) \) are parameters of the beam, which we define below. The corresponding time-averaged intensity (or irradiance) distribution is

\[
I(r, z) = \frac{|E(r, z)|^2}{2\eta} = I_0 \left( \frac{w_0}{w(z)} \right)^2 \exp\left( -\frac{2r^2}{w^2(z)} \right).
\]

where \( I_0 = I(0,0) \) is the intensity at the center of the beam at its waist. The constant \( \eta \) is the characteristic impedance of the medium in which the beam is propagating. For free space,

\[
\eta = \eta_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} = \frac{1}{\varepsilon_0 c} \approx 376.7 \, \Omega.
\]

\( r \) is the radial distance from the center axis of the beam, \( z \) is the axial distance from the beam's narrowest point (the "waist"), \( i \) is the imaginary unit (for which \( i^2 = -1 \)), \( \frac{2\pi}{\lambda} \) is the wave number (in radians per meter), \( E_0 = | E(0,0) | \), \( w(z) \) is the radius at which the field amplitude and intensity drop to \( 1/e \) and \( 1/e^2 \) of their axial values, respectively, and \( w_0 = w(0) \) is the waist size (described in more detail below).
**Beam parameters**

The geometry and behavior of a Gaussian beam are governed by a set of beam parameters, which are defined in the following sections.

For a Gaussian beam propagating in free space, the spot size $w(z)$ will be at a minimum value $w_0$ at one place along the beam axis, known as the beam waist. For a beam of wavelength $\lambda$ at a distance $z$ along the beam from the beam waist, the variation of the spot size is given by

$$w(z) = w_0 \sqrt{1 + \left( \frac{z}{z_R} \right)^2}.$$

where the origin of the z-axis is defined, without loss of generality, to coincide with the beam waist, and where

$$z_R = \frac{\pi w_0^2}{\lambda}$$

is called the Rayleigh range.

**Rayleigh range and confocal parameter**

At a distance from the waist equal to the Rayleigh range $z_R$, the width $w$ of the beam is

$$w(\pm z_R) = w_0 \sqrt{2}$$

The distance between these two points is called the confocal parameter or depth of focus of the beam:

$$b = 2z_R = \frac{2\pi w_0^2}{\lambda}.$$

**Radius of curvature**

$R(z)$ is the radius of curvature of the wavefronts comprising the beam. Its value as a function of position is

$$R(z) = z \left[ 1 + \left( \frac{z_R}{z} \right)^2 \right].$$

**Beam divergence**
The parameter \( w(z) \) approaches a straight line for \( z \gg z_R \). The angle between this straight line and the central axis of the beam is called the *divergence* of the beam. It is given by

\[
\theta \sim \frac{\lambda}{\pi w_0} \quad (\theta \text{ in radians})
\]

The total angular spread of the beam far from the waist is then given by

\[
\Theta = 2\theta.
\]

Because of this property, a Gaussian laser beam that is focused to a small spot spreads out rapidly as it propagates away from that spot. To keep a laser beam very well *collimated*, it must have a large diameter. This relationship between beam width and divergence is due to *diffraction*. Non-Gaussian beams also exhibit this effect, but a Gaussian beam is a special case where the product of width and divergence is the smallest possible.

Since the gaussian beam model uses the paraxial approximation, it fails when wavefronts are tilted by more than about 30° from the direction of propagation\([1]\). From the above expression for divergence, this means the Gaussian beam model is valid only for beams with waists larger than about \( 2\lambda/\pi \).

Laser beam quality is quantified by the *beam parameter product* (BPP). For a Gaussian beam, the BPP is the product of the beam's divergence and waist size \( w_0 \). The BPP of a real beam is obtained by measuring the beam's minimum diameter and far-field divergence, and taking their product. The ratio of the BPP of the real beam to that of an ideal Gaussian beam at the same wavelength is known as \( M^2 \) ("M squared"). The \( M^2 \) for a Gaussian beam is one. All real laser beams have \( M^2 \) values greater than one, although very high quality beams can have values very close to one.

**References**


- [Gaussian Beam Propagation](#) - CVI Melles Griot Technical Guide
- [Gaussian Beam Optics Tutorial, Newport](#)